Deep Generative Models: GANs and VAE

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Split, Croatia 2017

UNIVERSITY OF AMSTERDAM

Do we need generative modeling?

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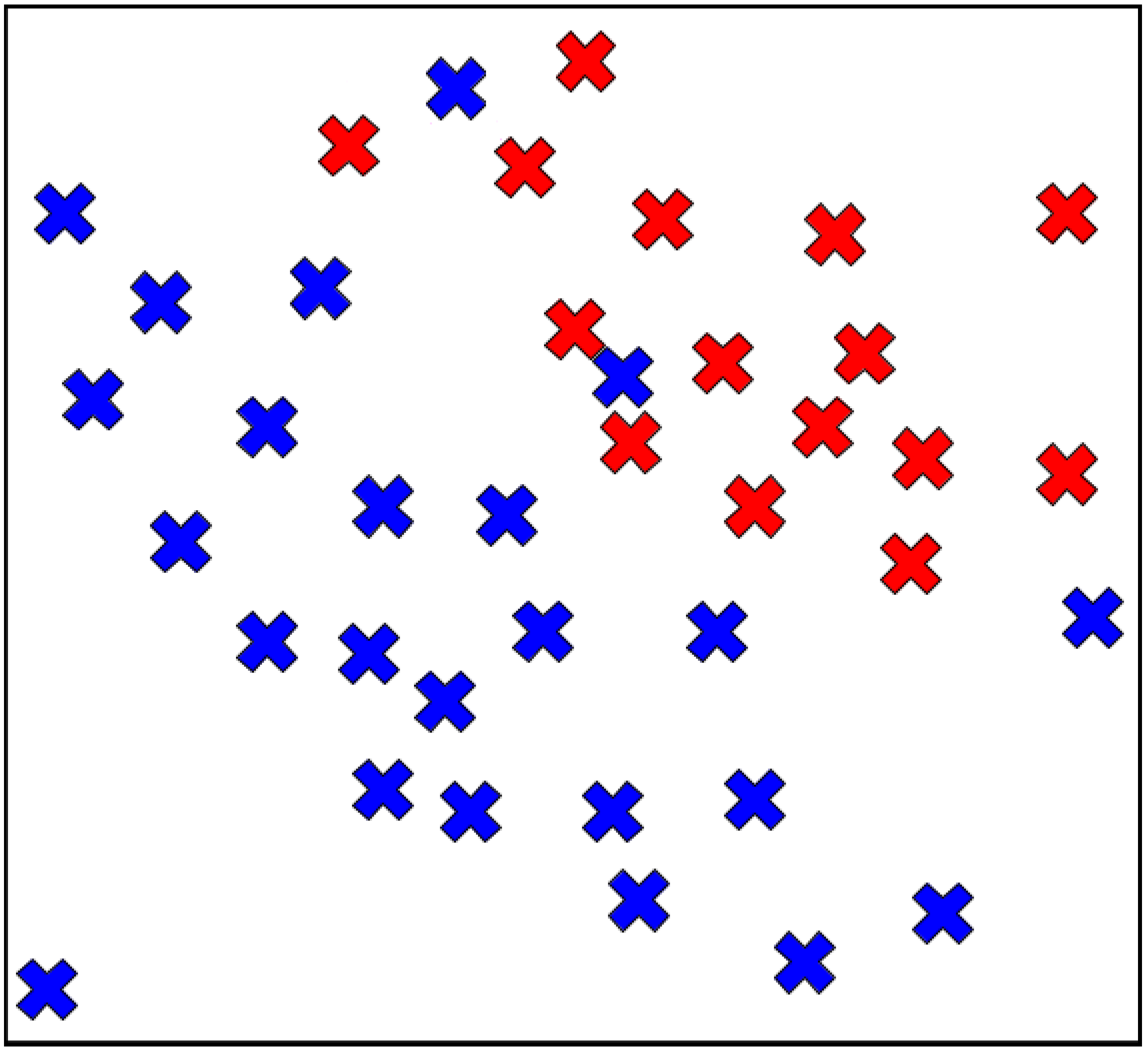

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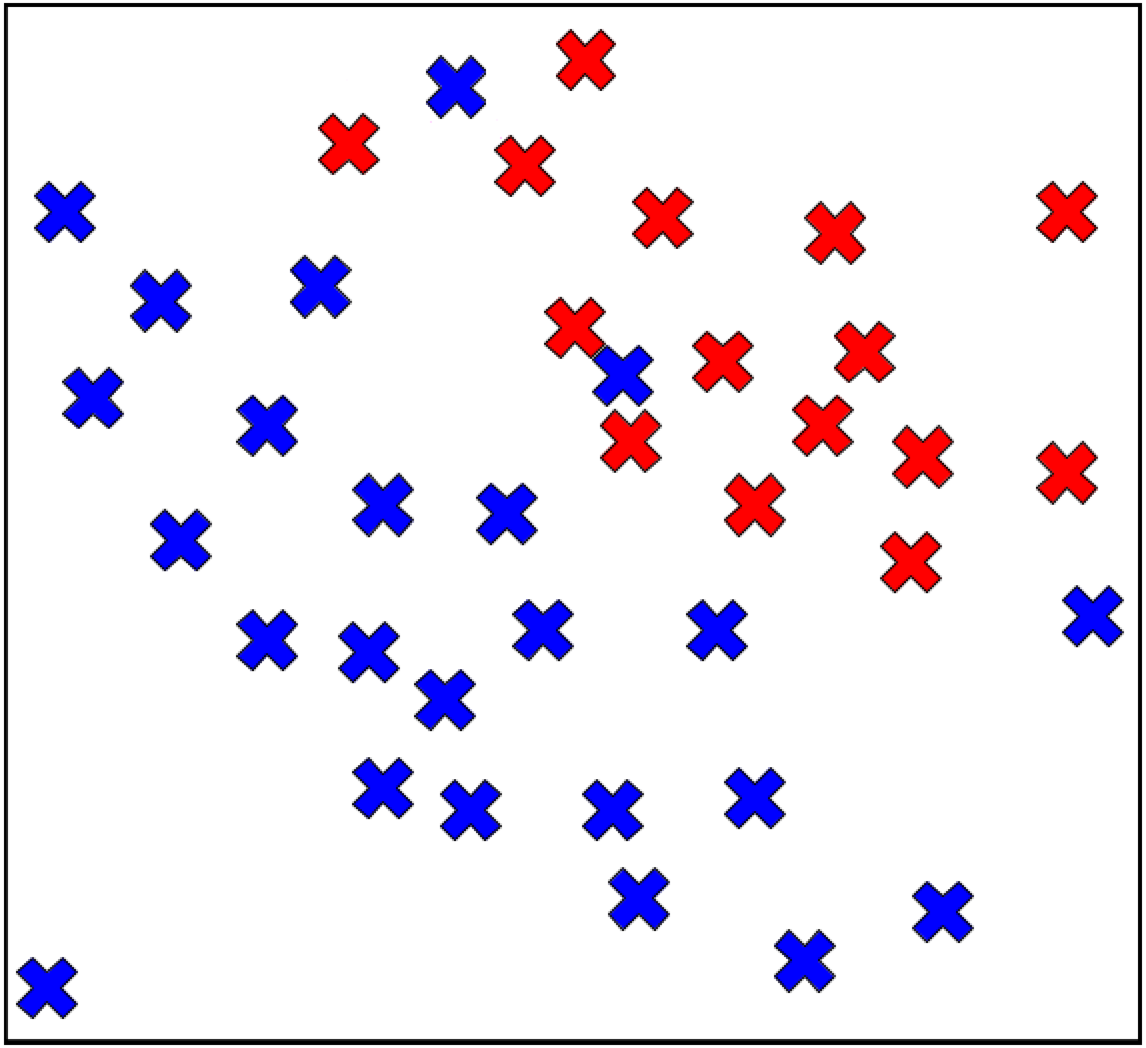

$p_\theta(y|x)$

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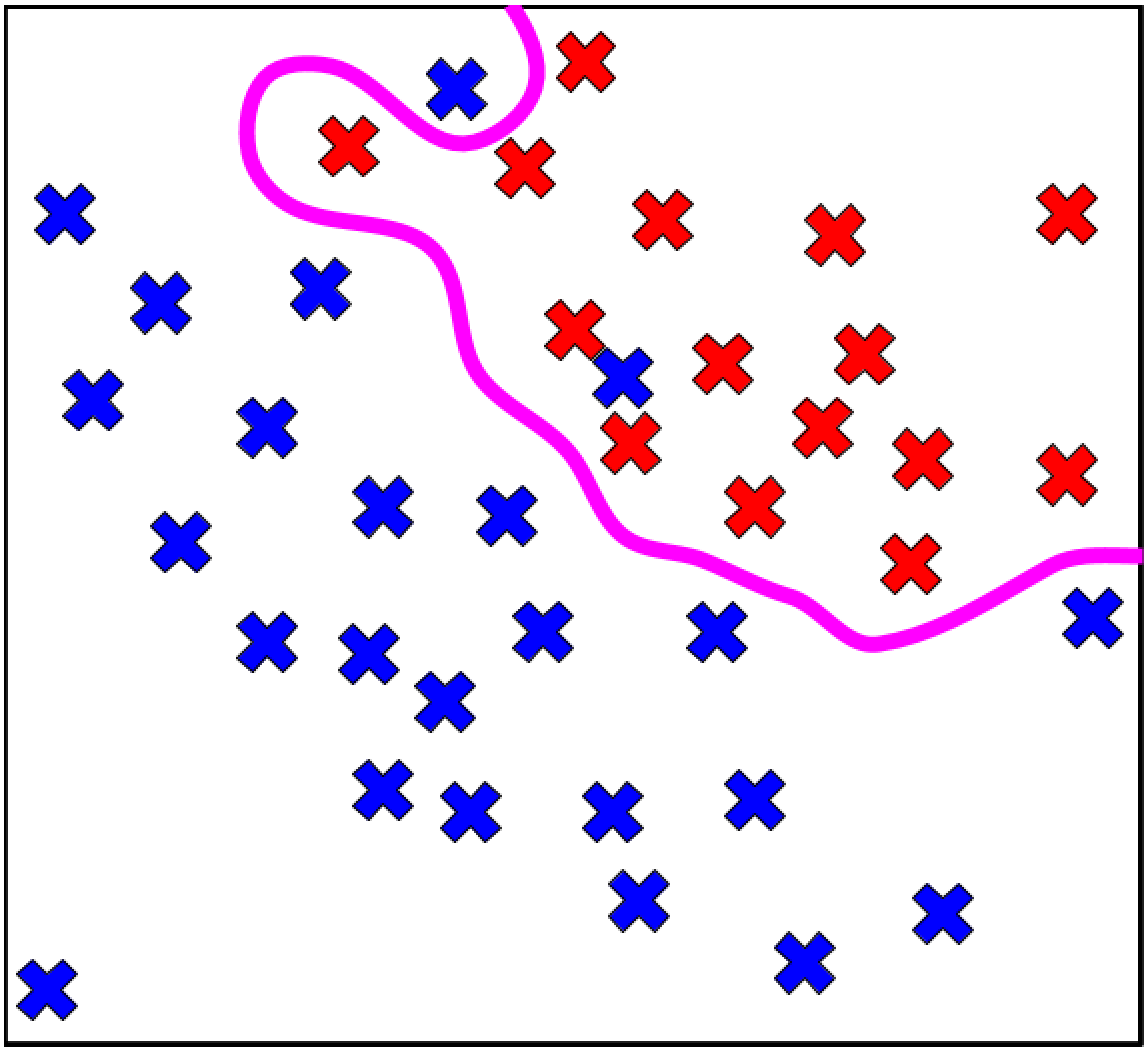

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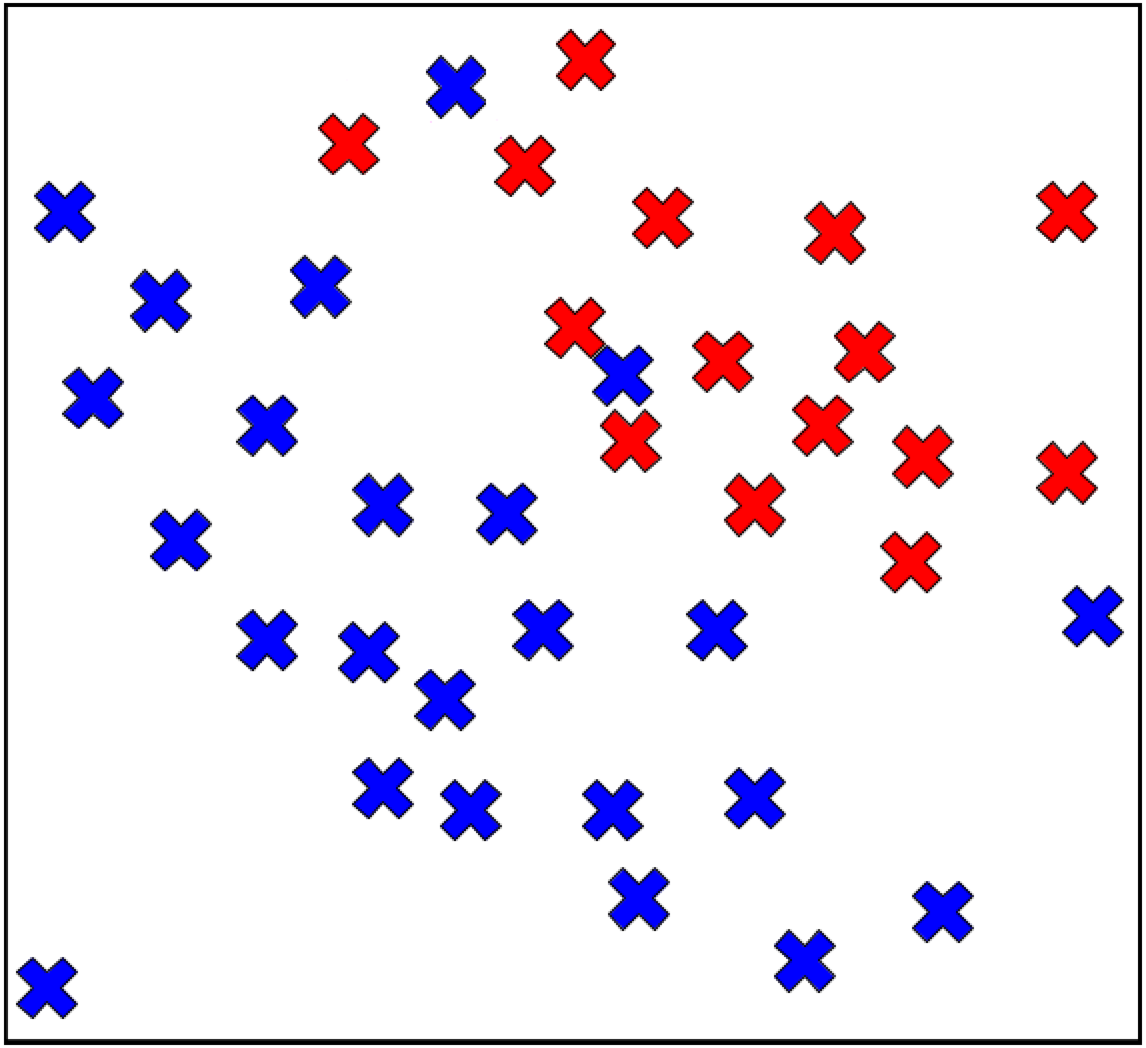

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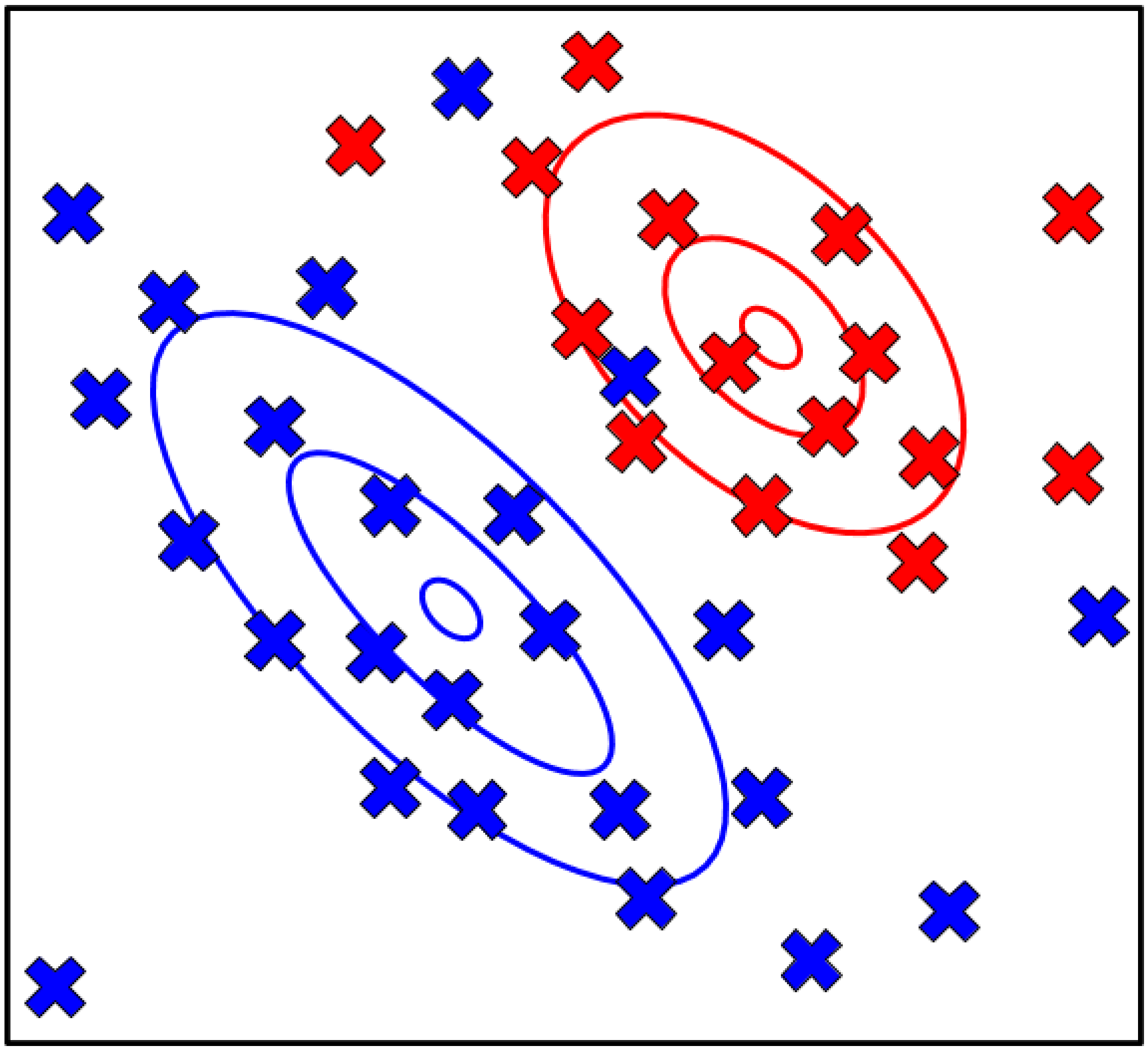

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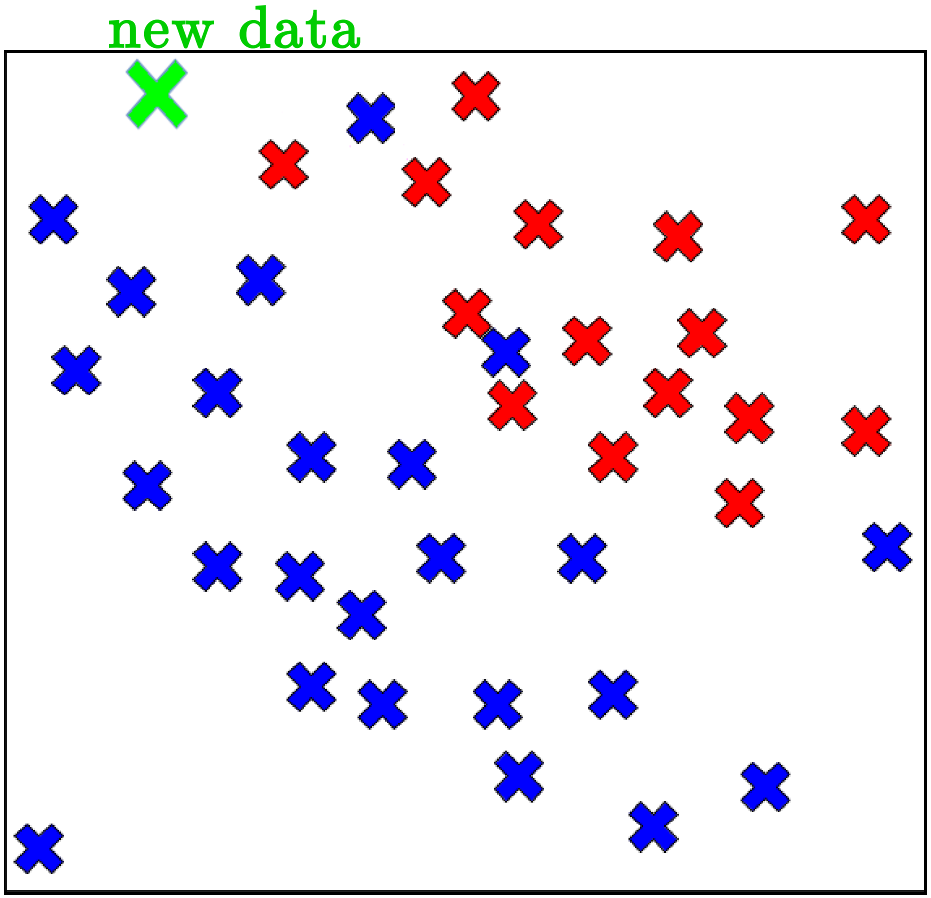

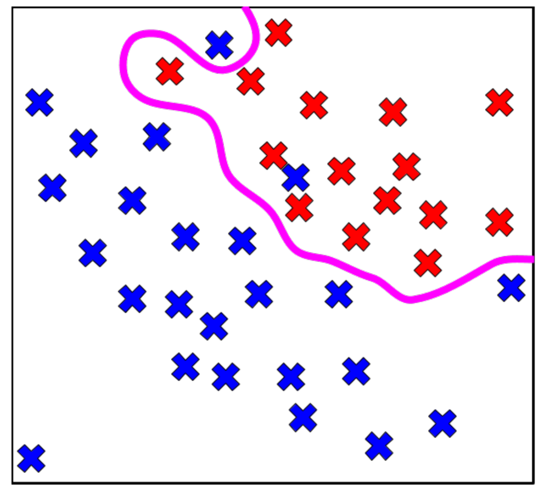

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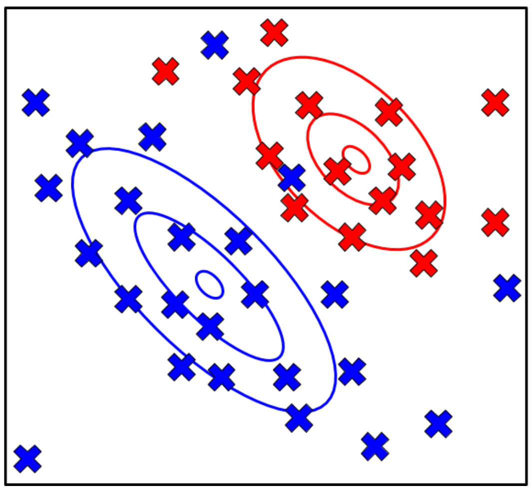

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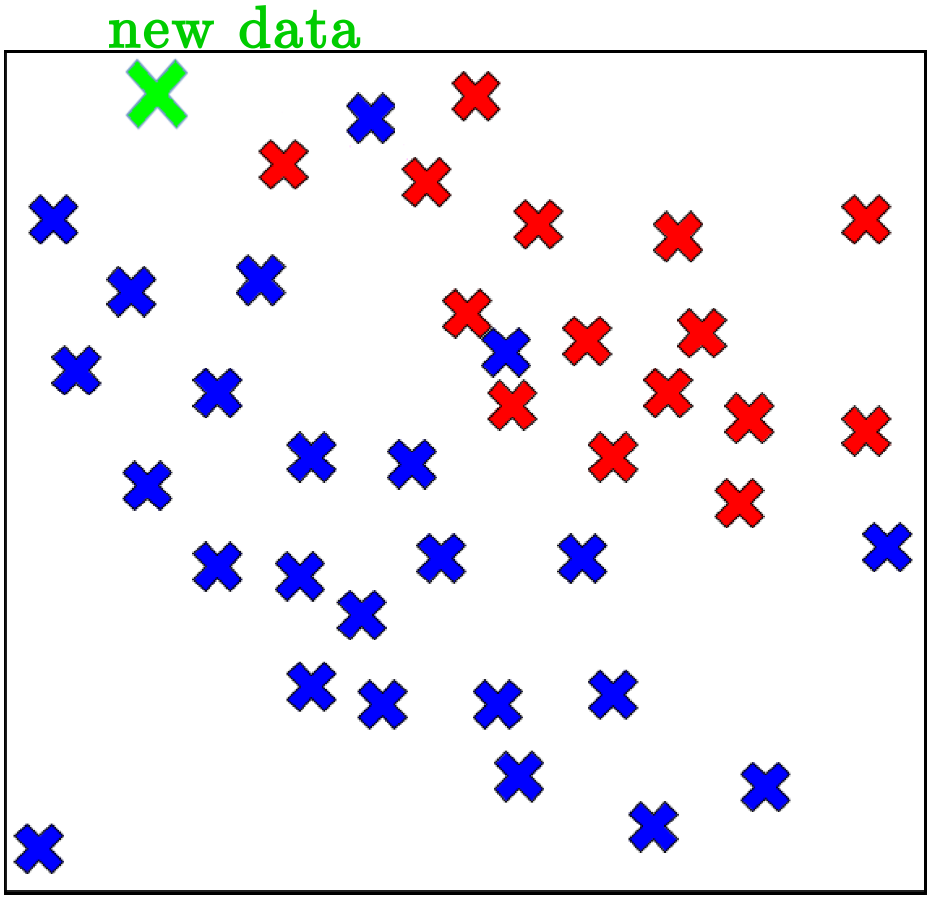

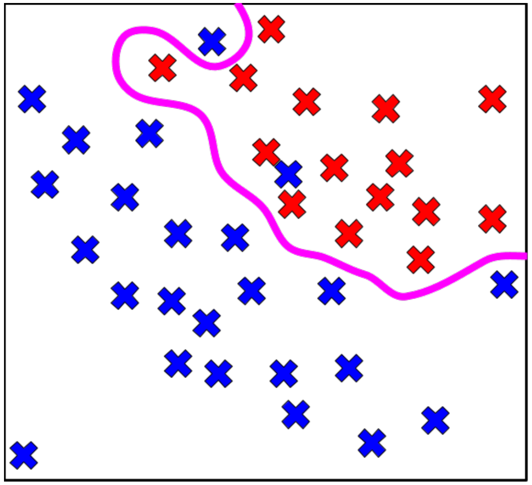

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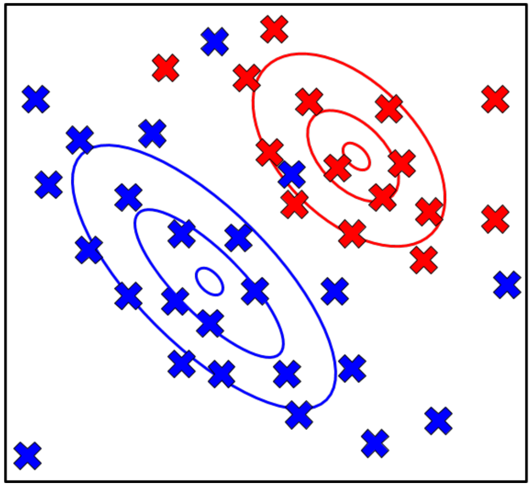

$p_\theta(x, y)$

High probability
of the **blue label**.
=
**Highly probable
decision!**

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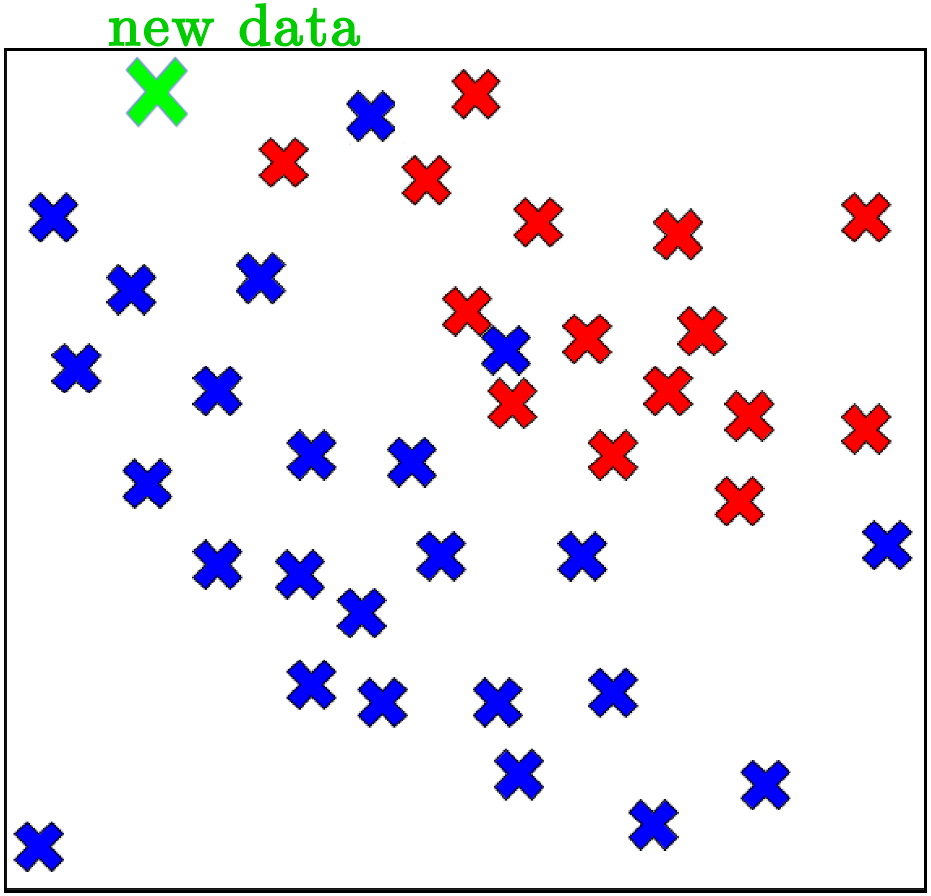

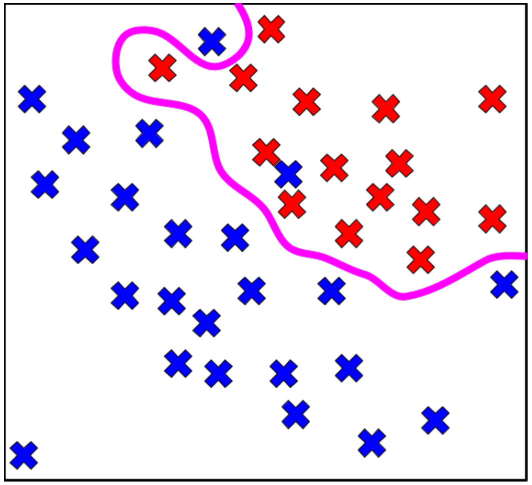

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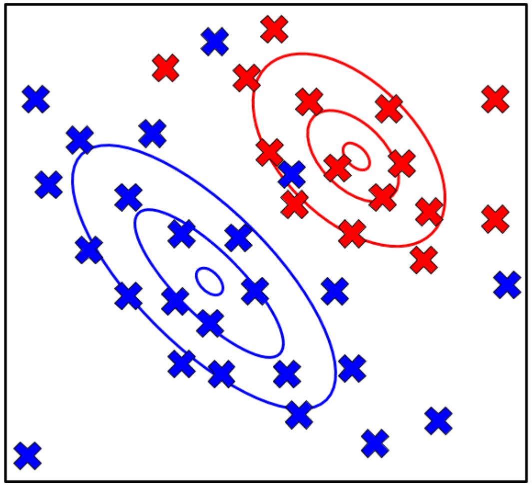


$p_\theta(x, y)$

High probability of the blue **label**.

$=$

Highly probable decision!

High probability of the blue **label**.

x

Low probability of the **object**.

$=$

Uncertain decision!

Generative Modeling

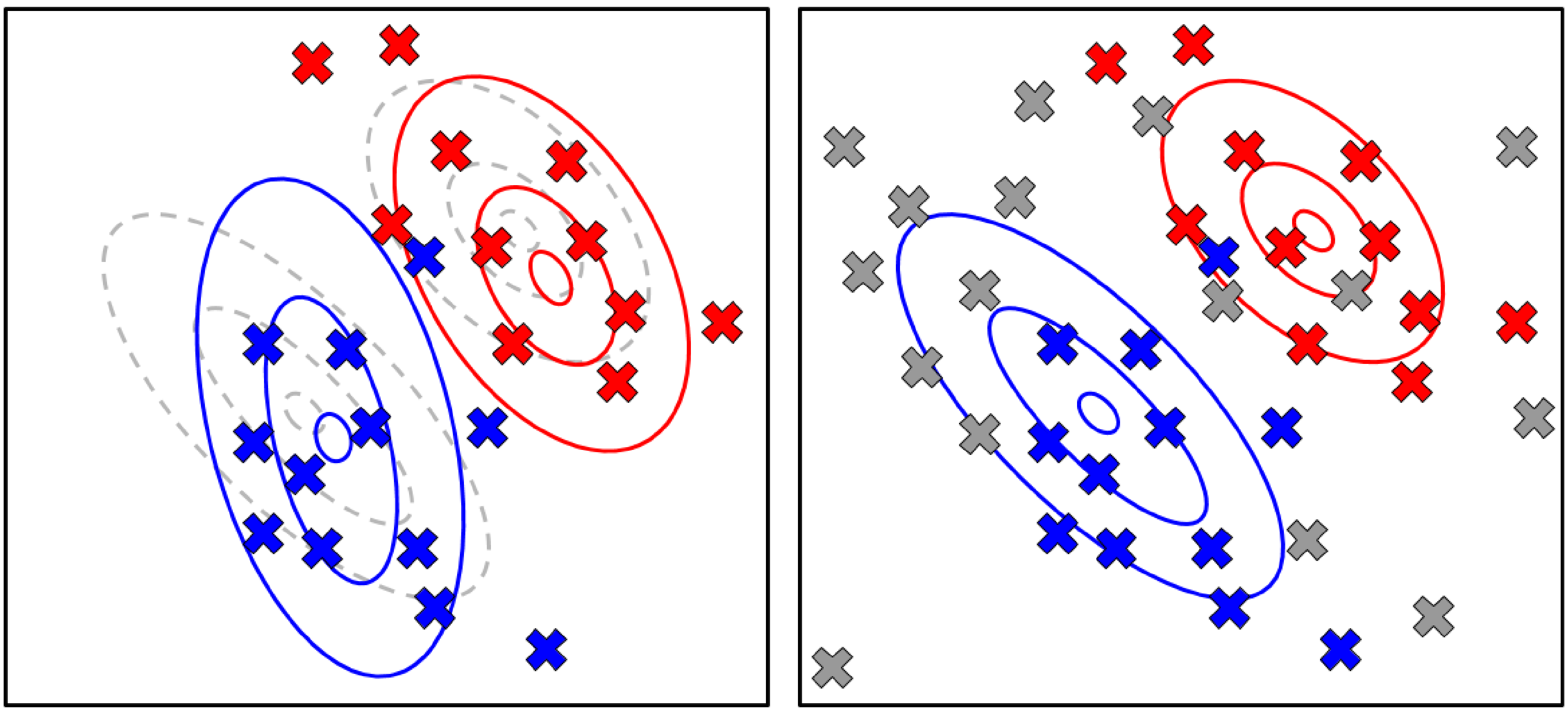
- Providing decision is not enough. *How to evaluate uncertainty? Distribution of y is only a part of the story.*

- Generalization problem. *Without knowing the distribution of $\mathbf{x}$ how we can generalize to new data?*

- Understanding the problem is crucial ("**What I cannot create, I do not understand**", Richard P. Feynman). *Properly modeling data is essential to make better decisions.*

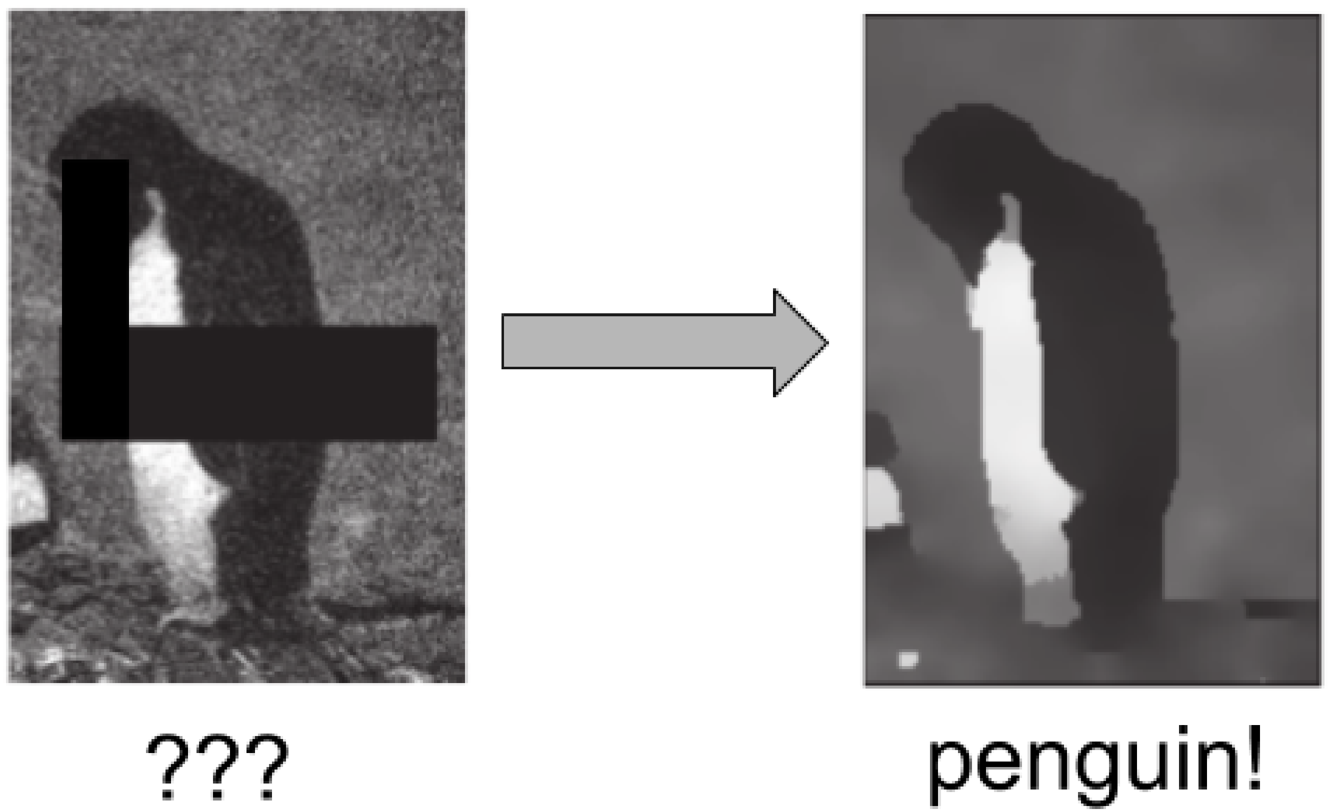
Generative Modeling

- Semi-supervised learning.
  Use unlabeled data to train a better classifier.



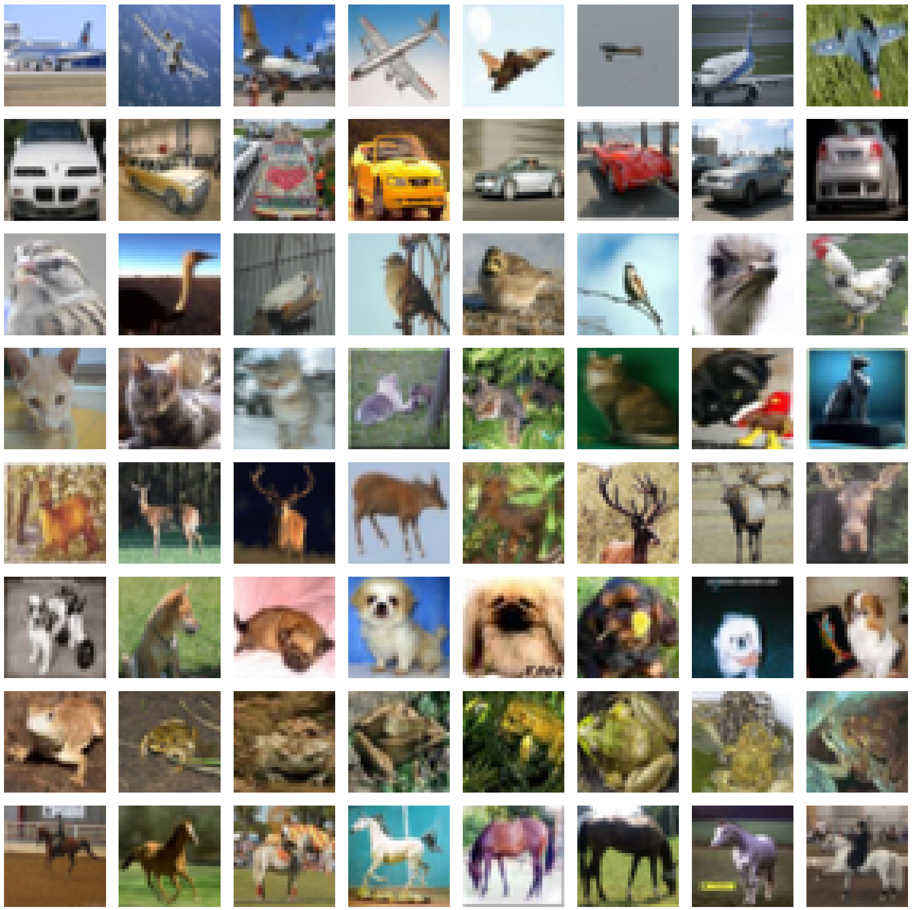
Generative Modeling

- Handling missing or distorted data.
  Reconstruct and/or denoise data.

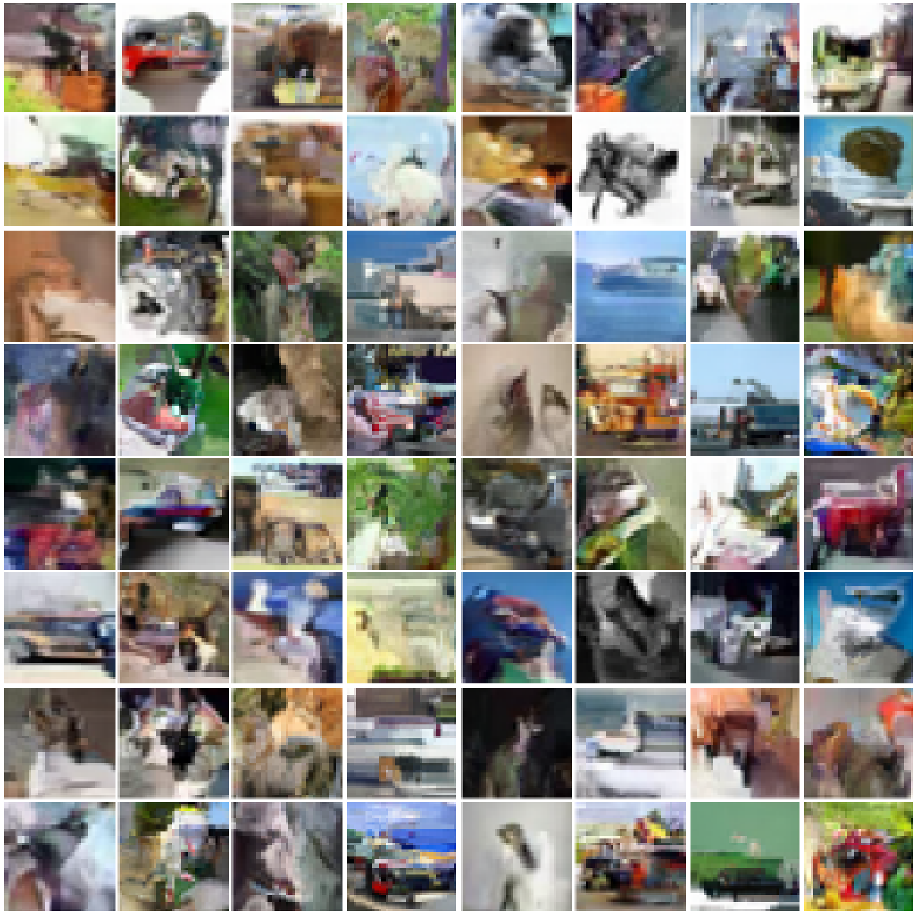


Generative Modeling

Image generation



Real



Generated

CHEN, Xi, et al. Variational lossy autoencoder. arXiv preprint arXiv:1611.02731, 2016.

Generative Modeling

Sequence generation

he had been unable to conceal the fact that there was a logical explanation for his inability to alter the fact that they were supposed to be on the other side of the house .
with a variety of pots strewn scattered across the vast expanse of the high ceiling , a vase of colorful flowers adorned the tops of the rose petals littered the floor and littered the floor .
atop the circular dais perched atop the gleaming marble columns began to emerge from atop the stone dais, perched atop the dais .

Generated

BOWMAN, Samuel R., et al. Generating sentences from a continuous space. arXiv preprint arXiv:1511.06349, 2015.

How to formulate a generative model?

Modeling in high-dimensional space is difficult.

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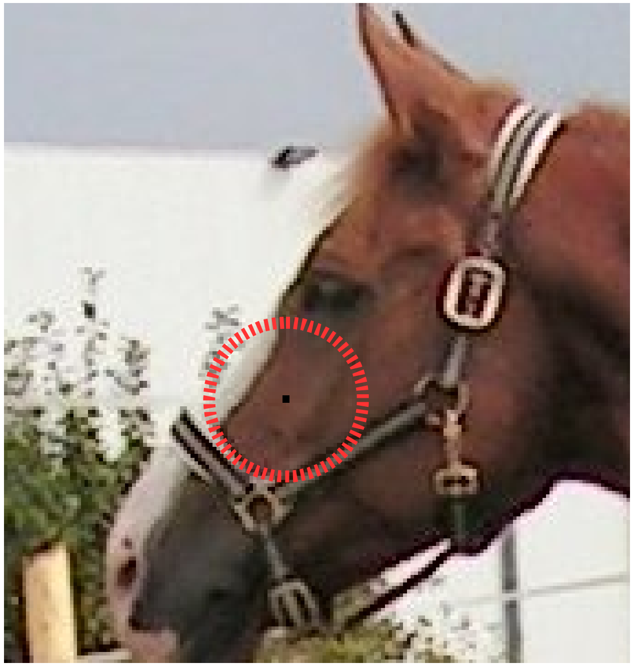

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Modeling in high-dimensional space is difficult.

$\rightarrow$ modeling all dependencies among pixels.

$$p(x) = \prod_{d=1}^{c} \psi_c(x_c)$$

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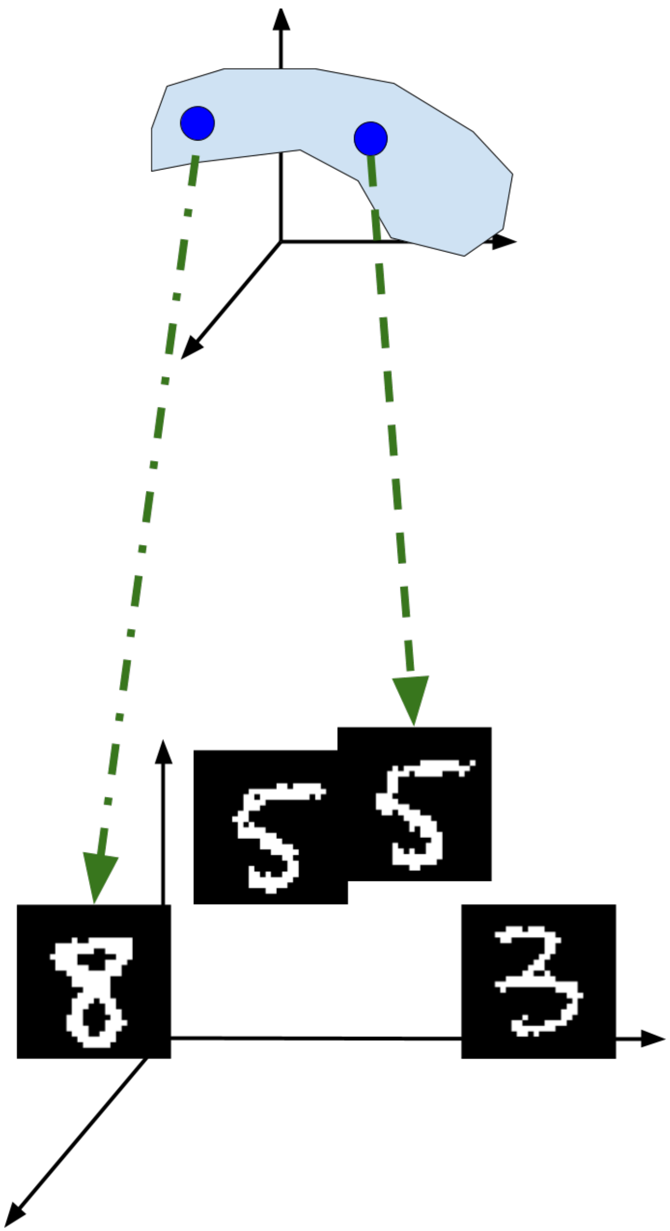
A possible **solution**? $\rightarrow$ **Latent variable models**

Latent Variable Models

- Latent variable model:

$$p(\mathbf{x}) = \int p_\theta(\mathbf{x}|\mathbf{z})\ p_\lambda(\mathbf{z})\ \mathrm{d}\mathbf{z}$$



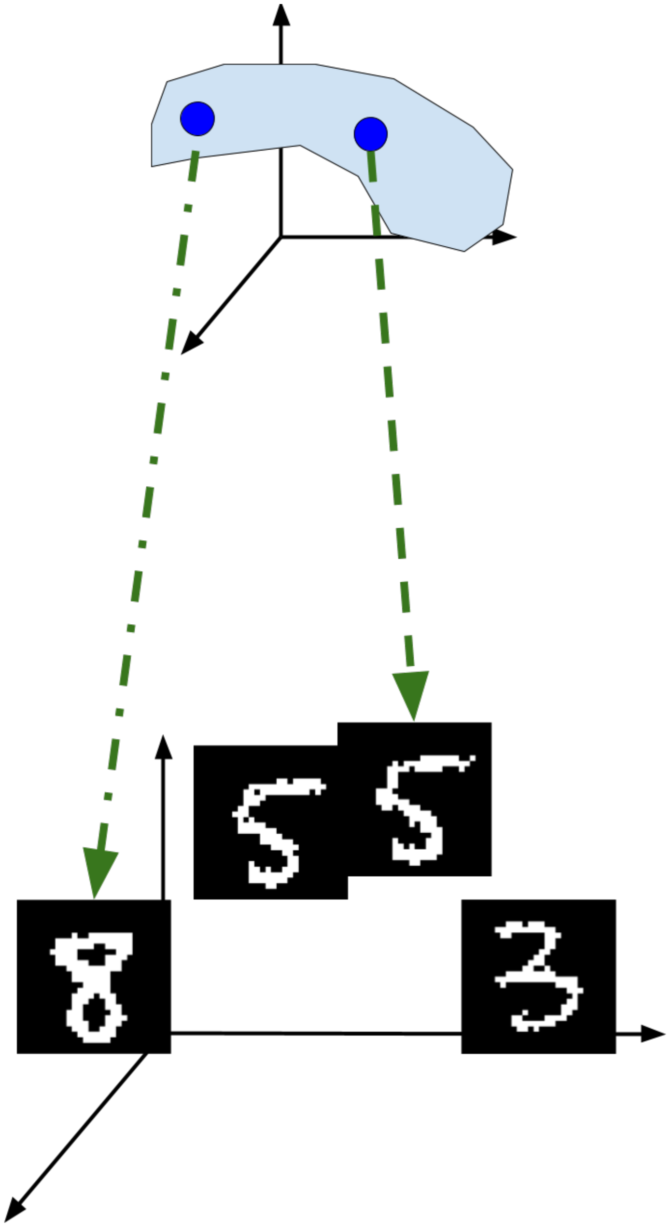
Latent Variable Models

- Latent variable model:

$$p(\mathbf{x}) = \int p_\theta(\mathbf{x}|\mathbf{z})\, p_\lambda(\mathbf{z})\; \mathrm{d}\mathbf{z}$$

First sample $\mathbf{z}$.

Second, sample $\mathbf{x}$ for given $\mathbf{z}$.



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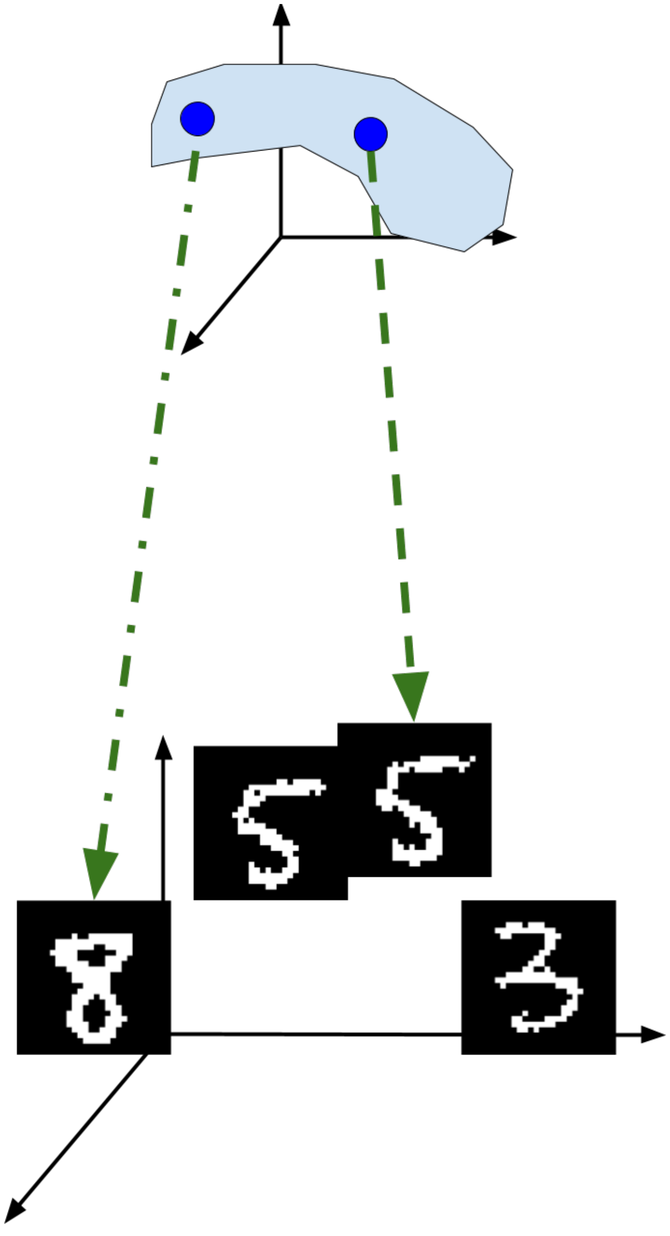

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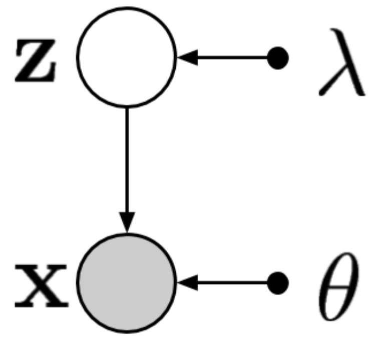

$$p(\mathbf{x}) = \int p_\theta(\mathbf{x}|\mathbf{z})\ p_\lambda(\mathbf{z})\ \mathrm{d}\mathbf{z}$$

- If $p_\theta(\mathbf{x}|\mathbf{z}) = \mathcal{N}(\mathbf{W}\mathbf{z} + \mathbf{b}, \Psi)$ and $p_\lambda(\mathbf{z}) = \mathcal{N}(\mu_0, \Sigma_0)$, then $\rightarrow$ **Factor Analysis**.

- What if we take a **non-linear transformation** of z? $\rightarrow$ ***an infinite mixture of Gaussians.***

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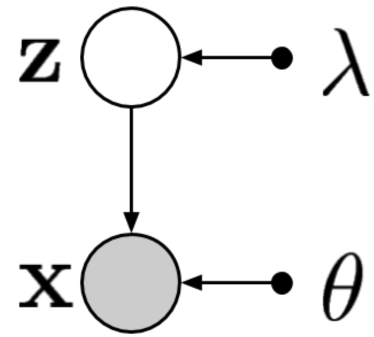

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Convenient but limiting!

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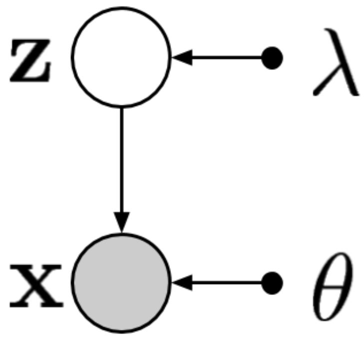

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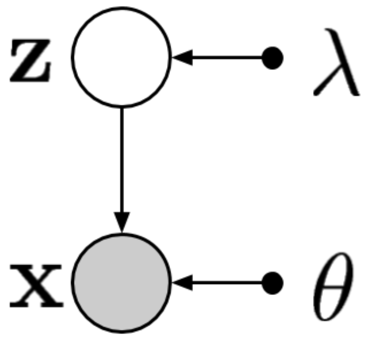

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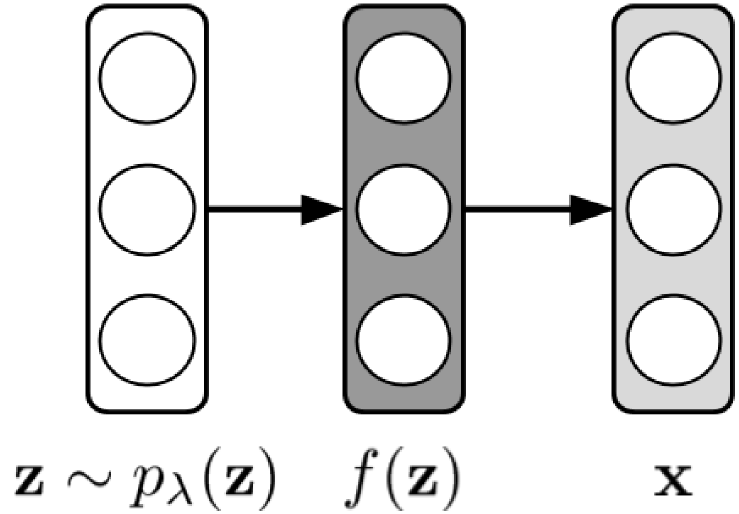
Neural network

Deep Generative Models (DGM): Density Network



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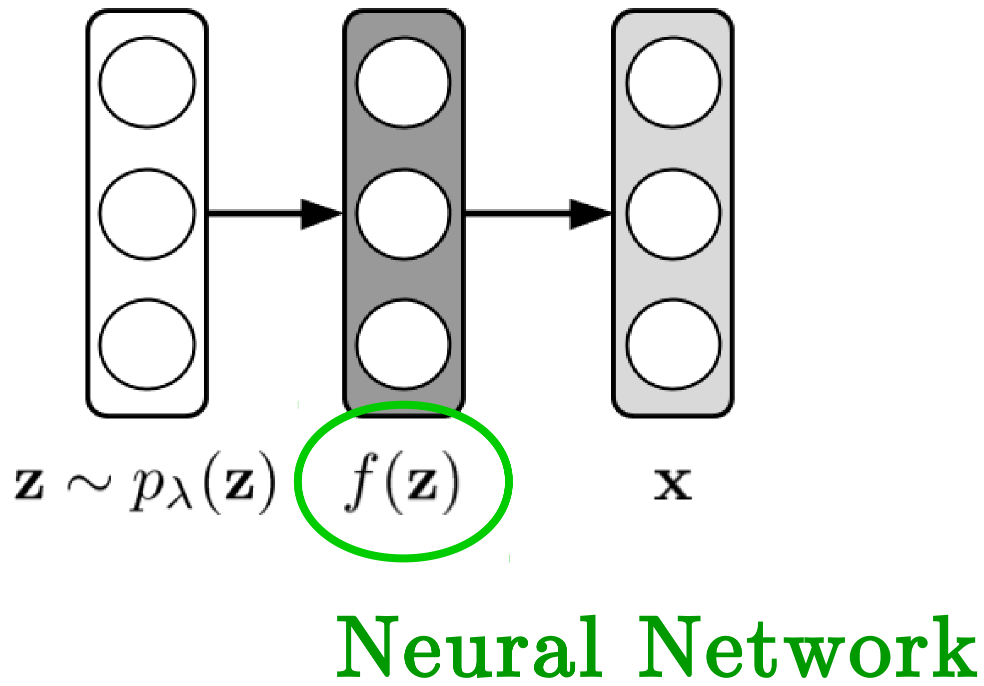
DGM: Density Network



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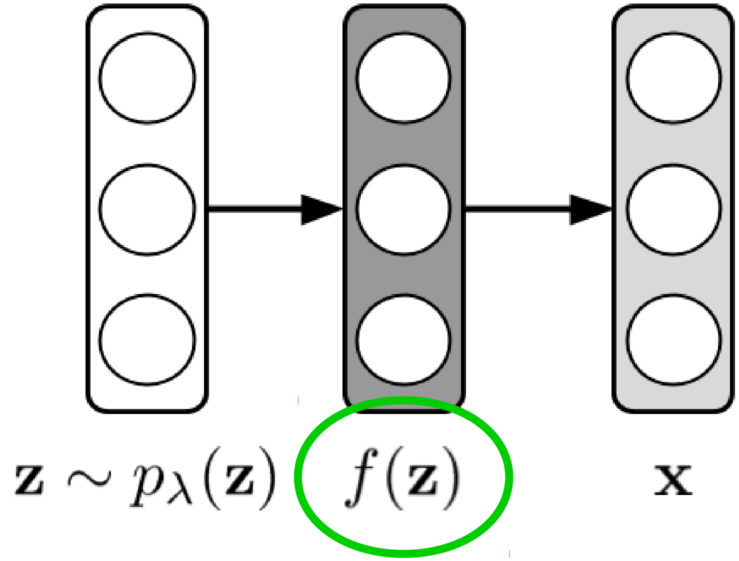

Neural Network

How to train this model?!

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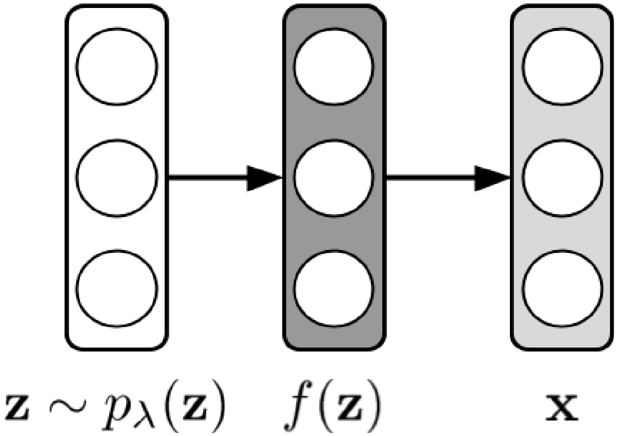

- MC approximation:

$$\log p(\mathbf{x}) = \log \int p_\theta(\mathbf{x}|\mathbf{z})\ p_\lambda(\mathbf{z})\ \mathrm{d}\mathbf{z}$$

$$\approx \log \frac{1}{S} \sum_{s=1}^{S} \exp\Big(\log p_\theta(\mathbf{x}|\mathbf{z}_s)\Big)$$

where:

$$\mathbf{z}_s \sim p_\lambda(\mathbf{z})$$

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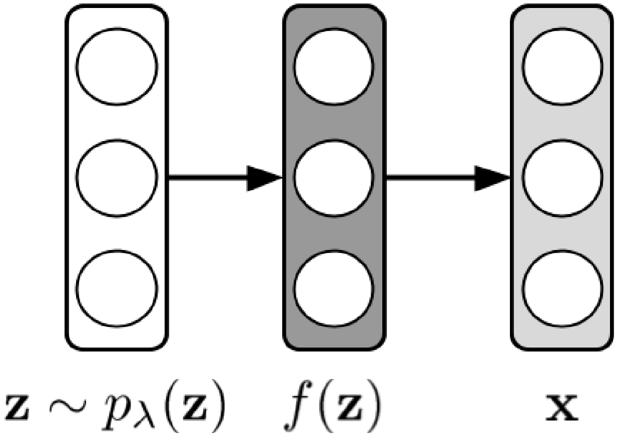

Sample $\mathbf{z}$ many times, apply log-sum-exp trick and maximize log-likelihood.

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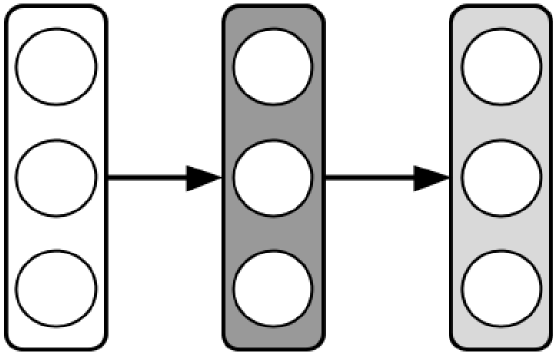

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It scales badly in high dimensional cases!

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DGM: Density Network

PROS

Log-**likelihood** approach

Easy **sampling**

Training using **gradient-based methods**

CONS

Requires **explicit** models

Fails in **high dim. cases**

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Can we do better?

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DGM: Generative Adversarial Nets

Let image two agents:

Goodfellow, I., et al. (2014). *Generative adversarial nets*. In Advances in neural information processing systems

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A fraud

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A fraud

... and a real artist

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The expert assesses a painting and gives her opinion.

An art expert

The fraud learns and tries to fool the expert.

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Hmmm... fake!

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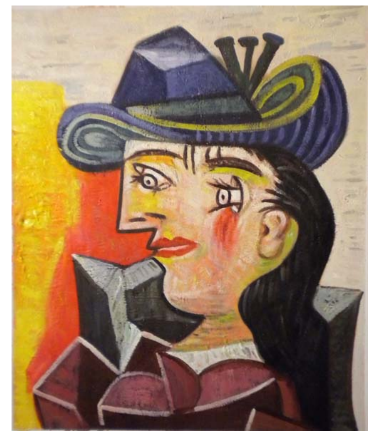

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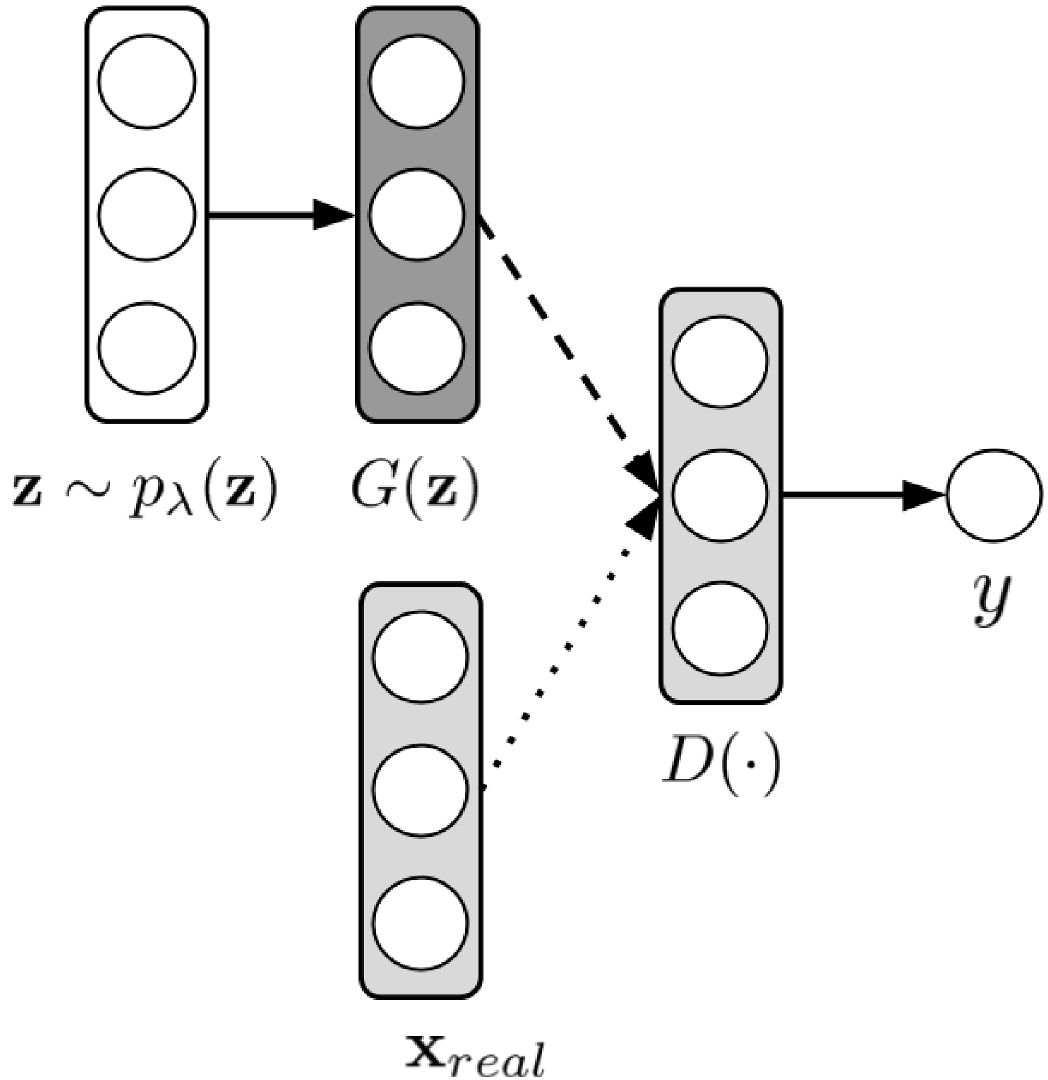

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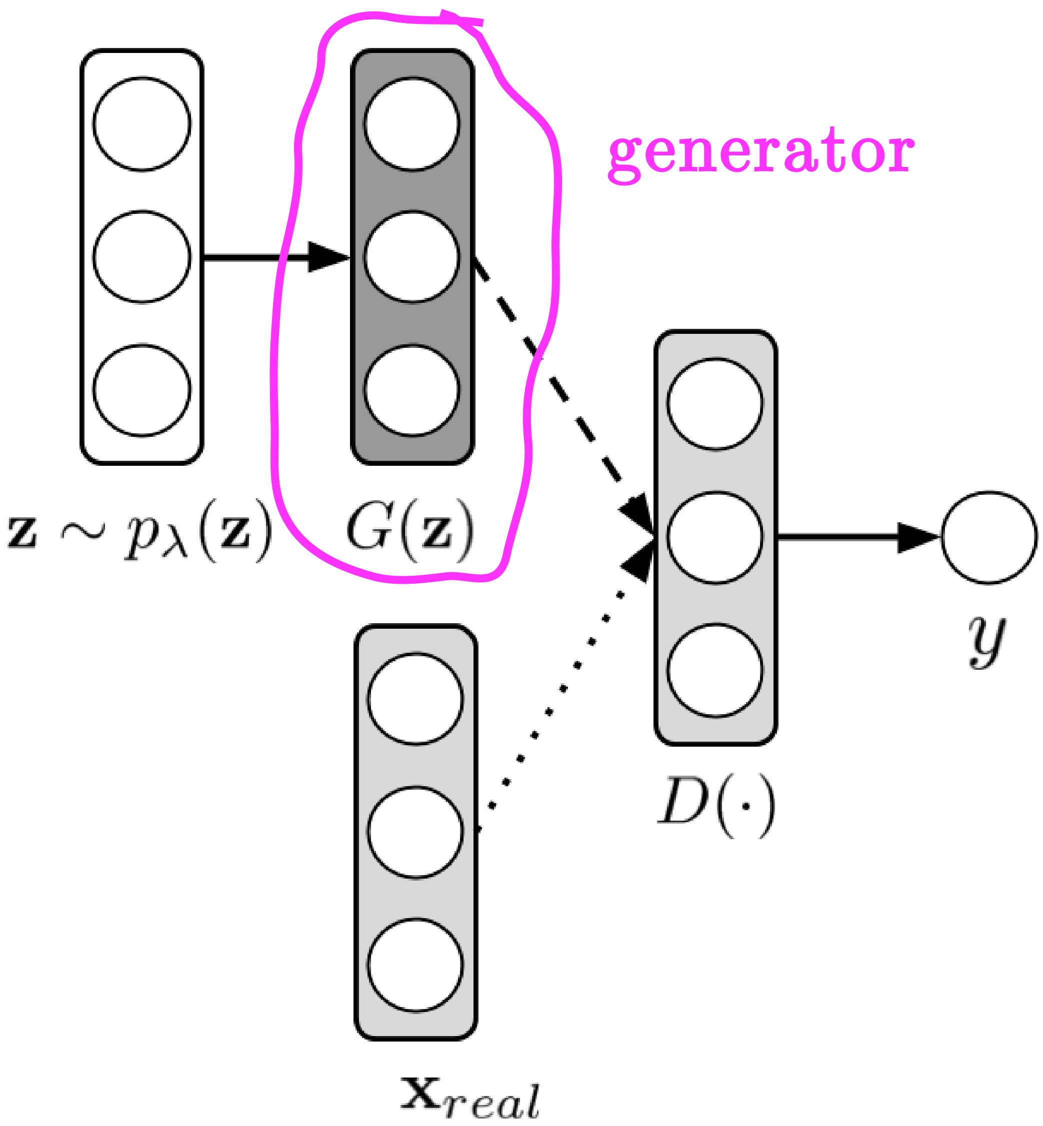

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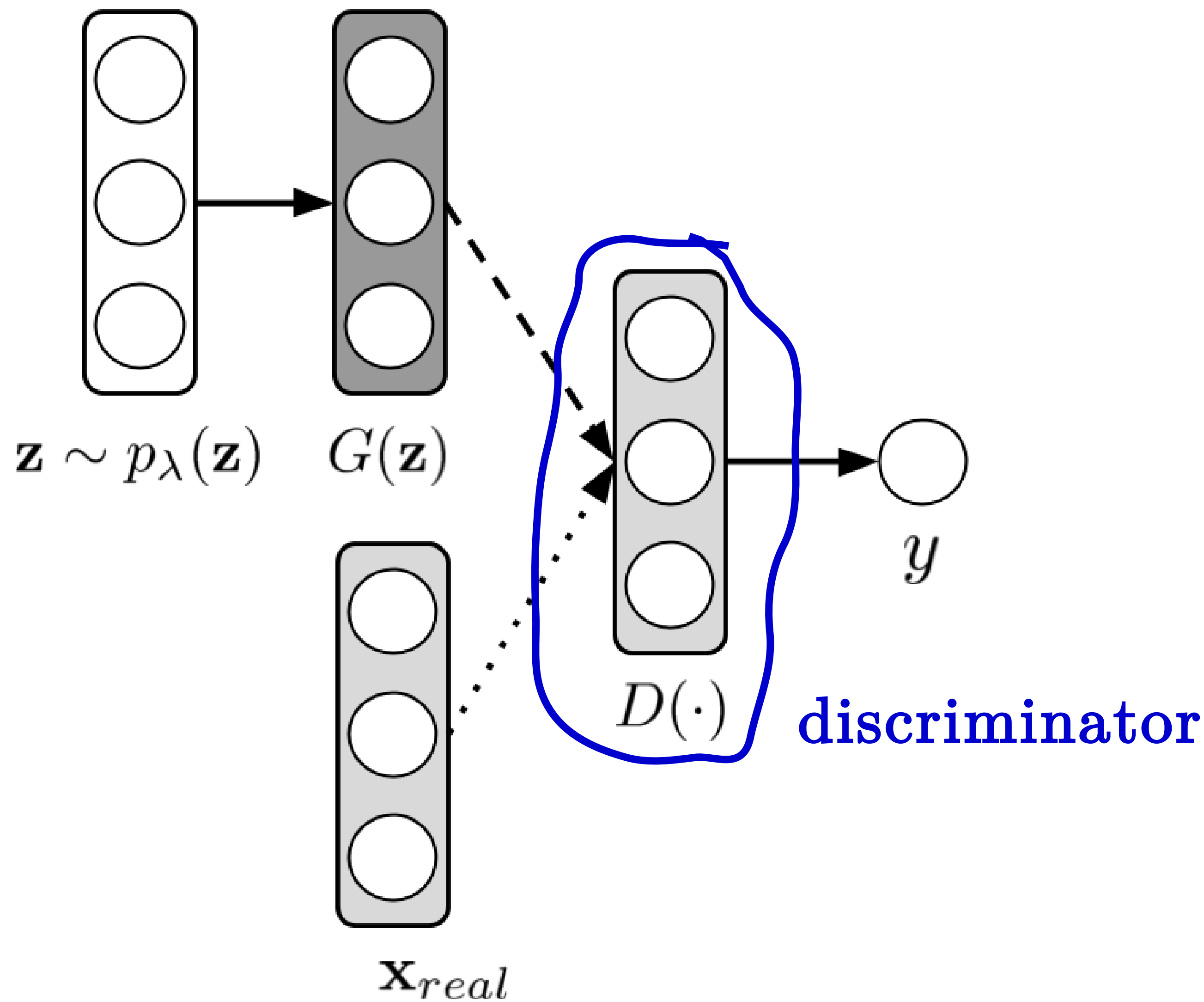

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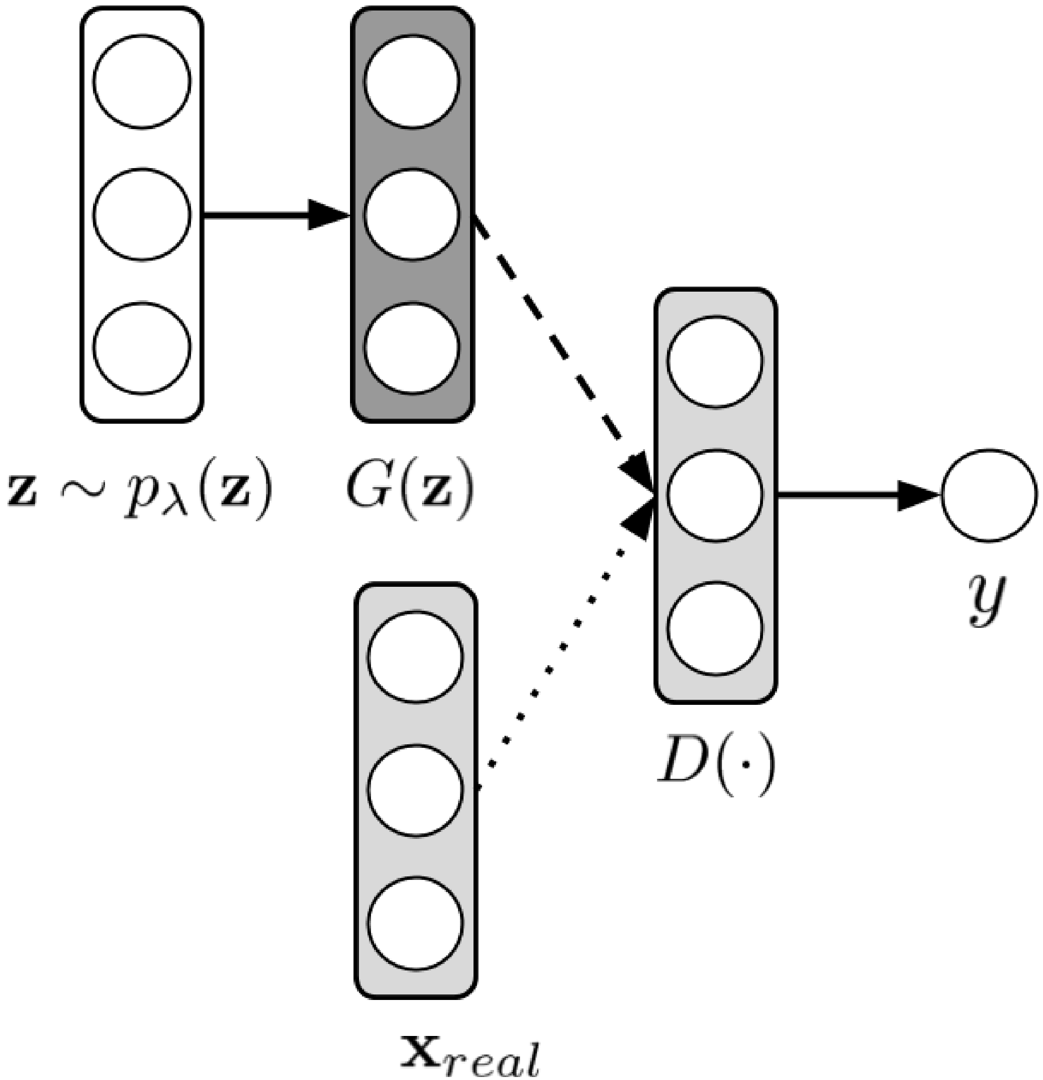
DGM: Generative Adversarial Nets

1. Sample **z**.

2. Generate $G(\mathbf{z})$.

3. Discriminate whether given image is **real** or **fake**.



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DGM: Generative Adversarial Nets

Formally, the problem is the following:

$$\min_G \max_D \mathbb{E}_{\mathbf{x}\sim p_{real}}[\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z}\sim p(\mathbf{z})}[\log(1 - D(G(\mathbf{z})))]$$

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Minimize wrt. generator

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Maximize wrt. discriminator

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Once we converge, we can generate images that are almost **indistinguishable** from real images.

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BUT training is very unstable...

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DGM: Generative Adversarial Nets

Training Data | Samples

Salimans, T., Goodfellow, I., Zaremba, W., Cheung, V., Radford, A., & Chen, X. (2016). Improved techniques for training GANs. In Advances in Neural Information Processing Systems (pp. 2234-2242).

DGM: GANs

PROS

Allows **implicit** models

Easy **sampling**

Training using **gradient-based methods**

Works in **high dim. cases**

CONS

Unstable training

Does not correspond to likelihood solution

No clear way for **quantitative assessment**

Missing mode problem

DGM: Wasserstein GAN

We can consider an earth-mover distance to formulate GAN-like optimization problem as follows:

$$\min_{G} \max_{D \in \mathcal{W}} \mathbb{E}_{\mathbf{x} \sim p_{real}} [D(\mathbf{x})] - \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} [D(G(\mathbf{z}))]$$

where the discriminator is a 1-Lipshitz function.

Arjovsky, M., Chintala, S., & Bottou, L. (2017). Wasserstein GAN. arXiv preprint arXiv:1701.07875.

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It means we need to **clip** weights of the discriminator, *i.e.*,

`clip(weights, -c, c)`.

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where the discriminator is a 1-Lipshitz function.

Wasserstein GAN **stabilizes training** (but other problems **remain**).

Arjovsky, M., Chintala, S., & Bottou, L. (2017). Wasserstein GAN. arXiv preprint arXiv:1701.07875.

DGM: More GANs (selected)

Deep convolutional generative adversarial networks

Radford, A., Metz, L., & Chintala, S. (2015). Unsupervised representation learning with deep convolutional generative adversarial networks. arXiv preprint arXiv:1511.06434.

Auxiliary classifier GANs

Odena, A., Olah, C., & Shlens, J. (2016). Conditional image synthesis with auxiliary classifier gans. arXiv preprint arXiv:1610.09585.

From optimal transport to generative modeling: the VEGAN cookbook

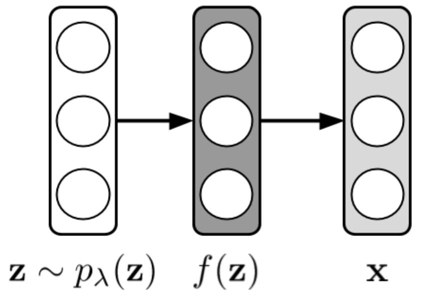
Bousquet, O., Gelly, S., Tolstikhin, I., Simon-Gabriel, C. J., & Schoelkopf, B. (2017). From optimal transport to generative modeling: the VEGAN cookbook. arXiv preprint arXiv:1705.07642.

Bidirectional Generative Adversarial Networks

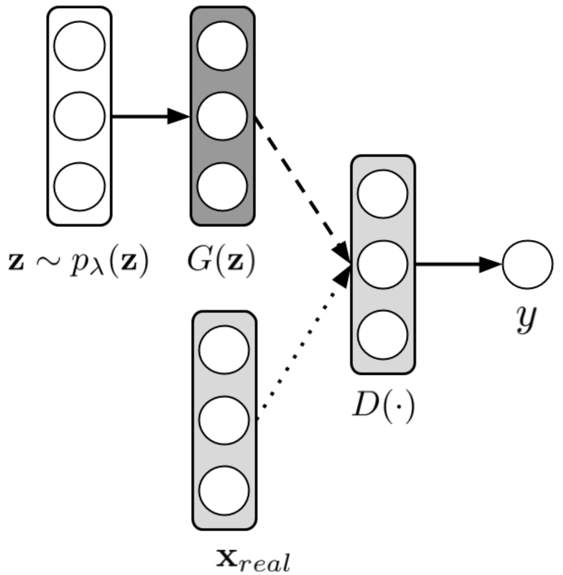
Donahue, J., Krähenbühl, P., & Darrell, T. (2016). Adversarial feature learning. arXiv preprint arXiv:1605.09782.

Questions?

DGM: so far we have

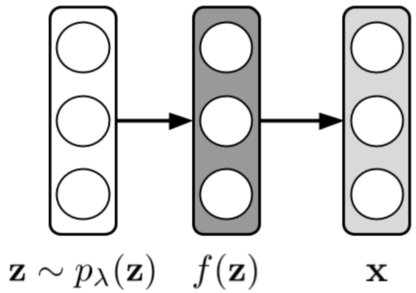


Density Network

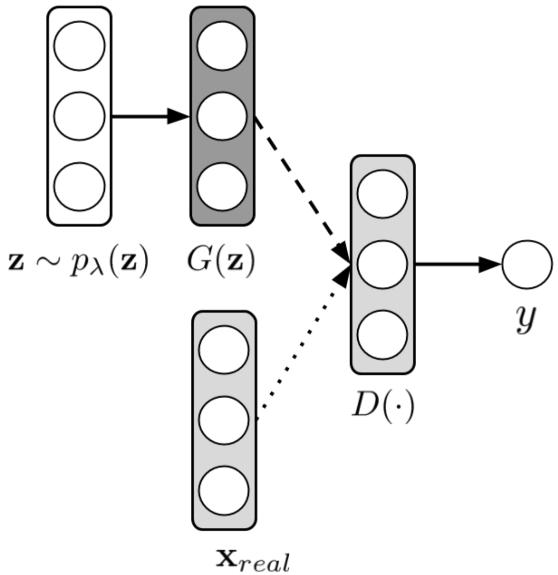


Generative Adversarial Net

DGM: so far we have



Density Network

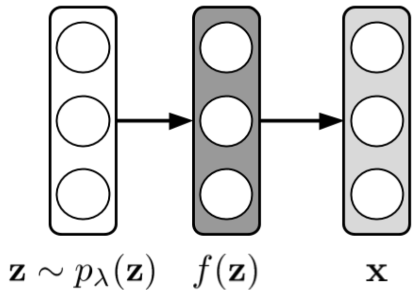


Generative Adversarial Net

Works only for low dim. cases...

Inefficient training...

DGM: so far we have



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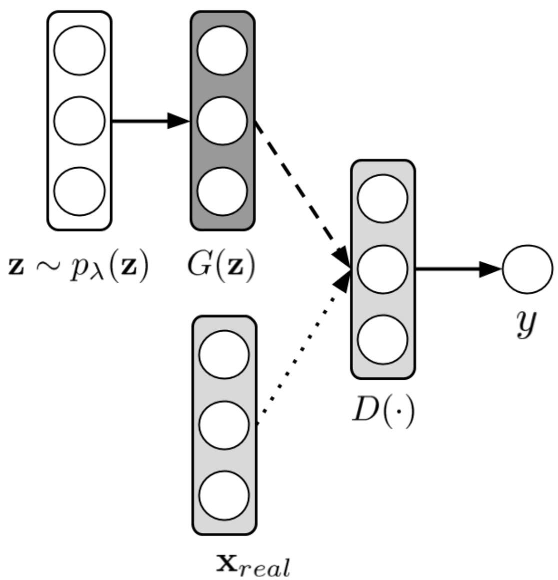
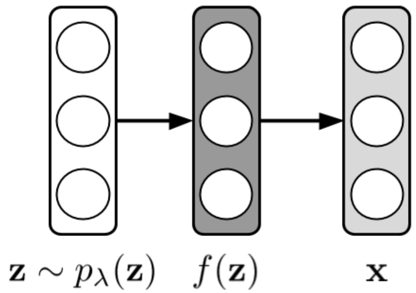


Generative Adversarial Net

Works for high dim. cases!

DGM: so far we have



Density Network

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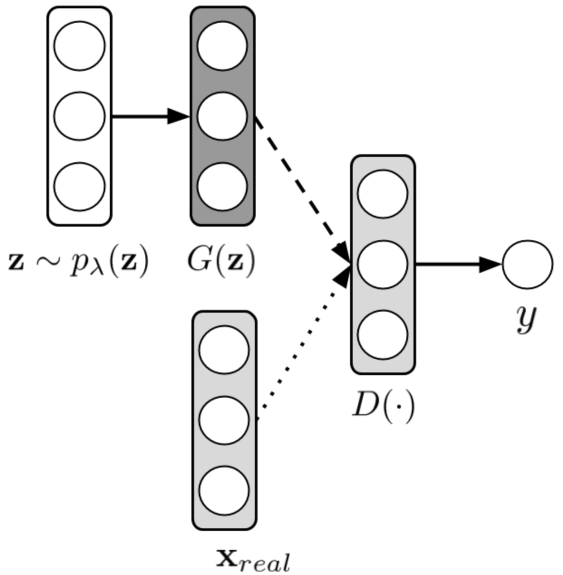


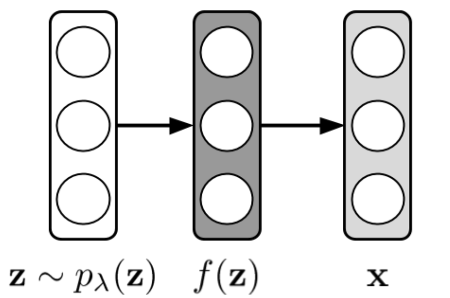
Generative Adversarial Net

Works for high dim. cases!

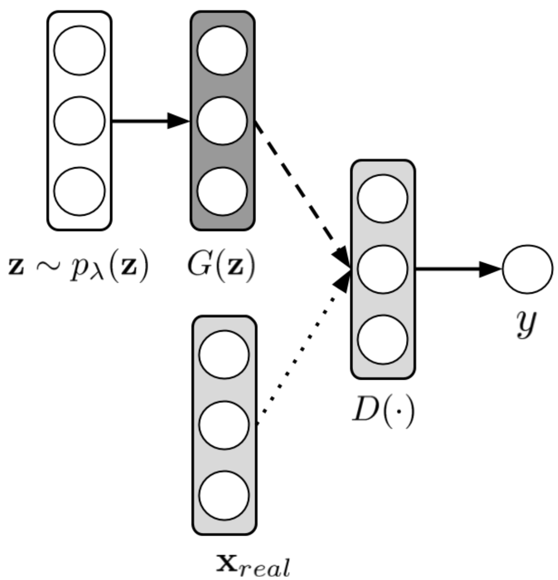
Doesn't train a distribution...

Unstable training...

DGM: so far we have



Density Network

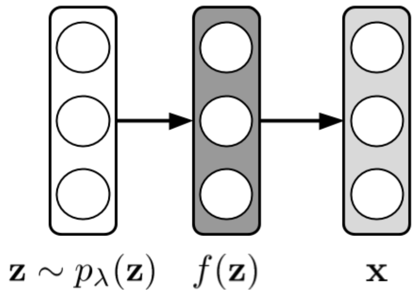


Generative Adversarial Net

QUESTION

Can we stick to the log-likelihood approach but with a simple training procedure?

DGM: so far we have



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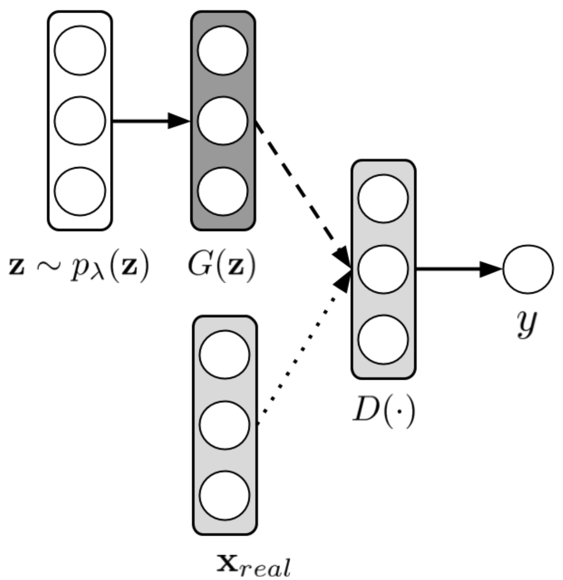
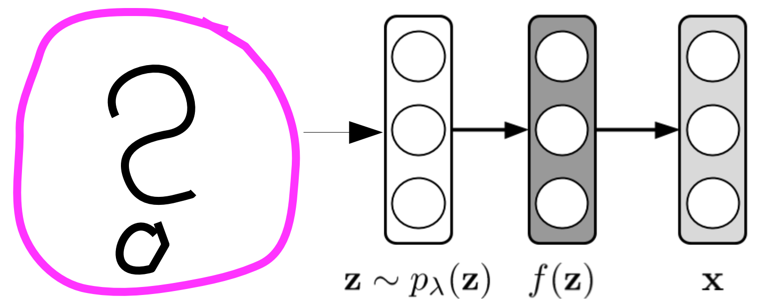
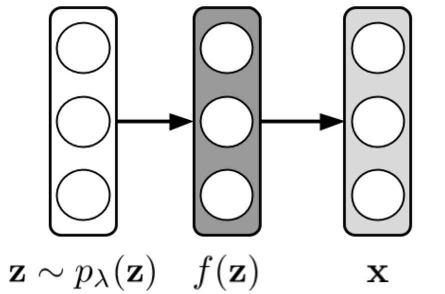


Generative Adversarial Net



DGM: Variational Auto-Encoder



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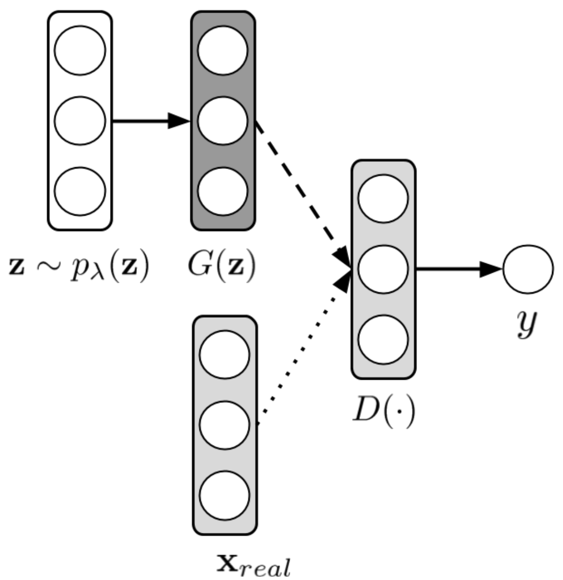

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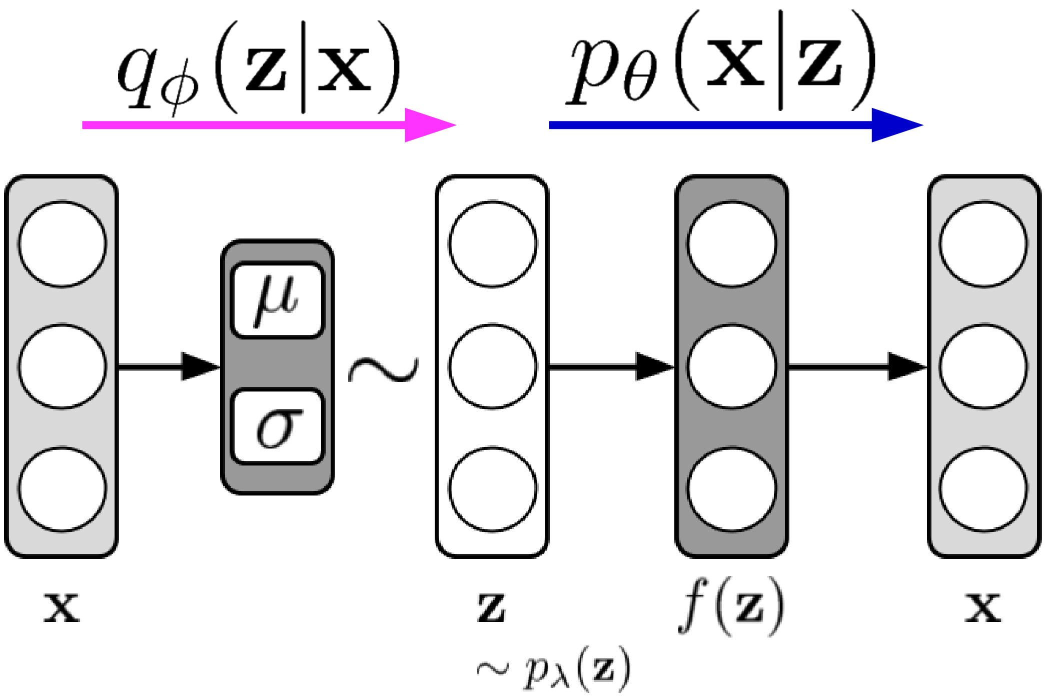

Variational Auto-Encoder

Kingma, D. P., & Welling, M. (2013). Auto-encoding Variational Bayes. arXiv preprint arXiv:1312.6114.

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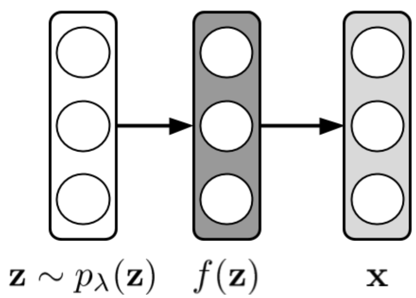

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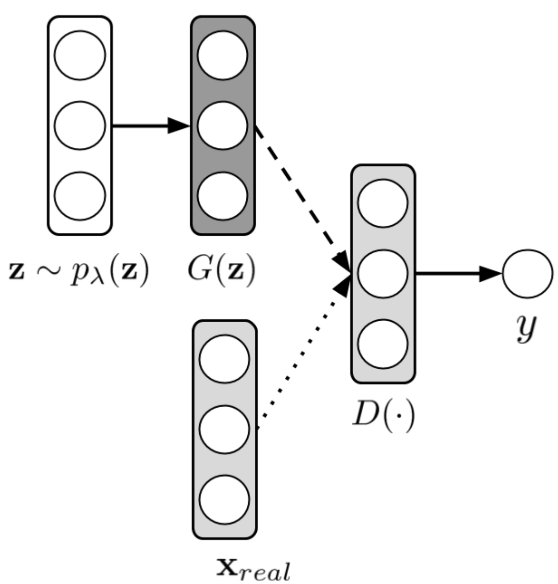

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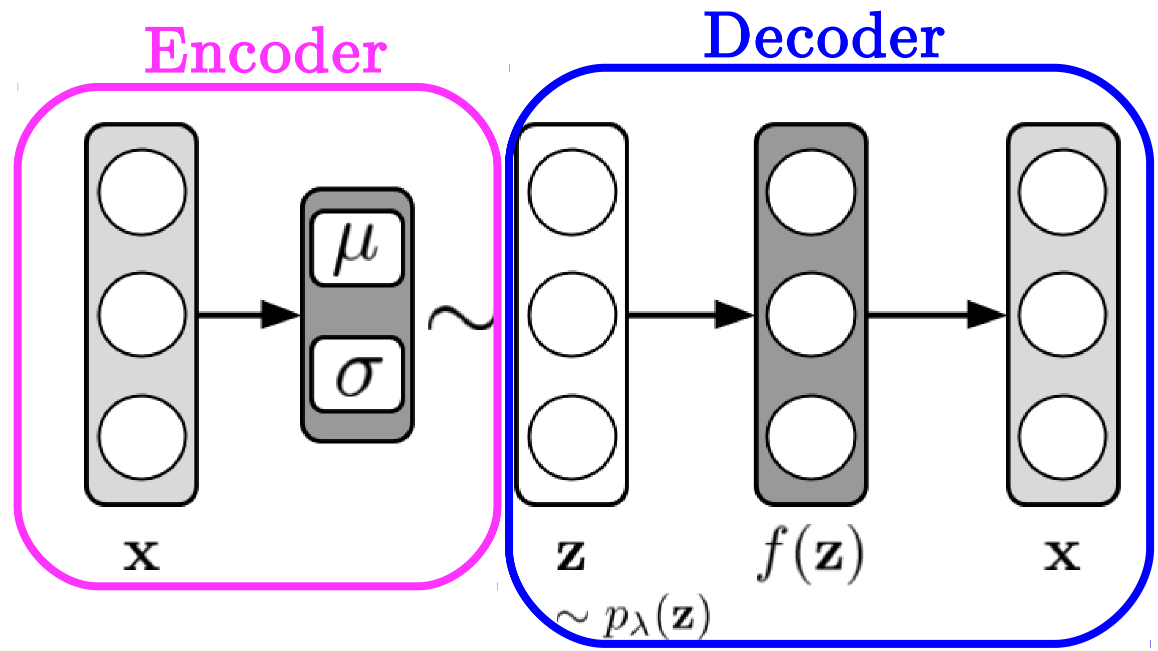

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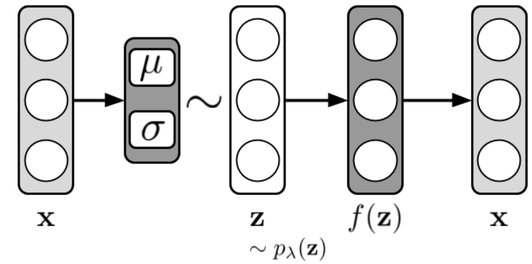

$$\begin{aligned}
\log p(\mathbf{x}) &= \log \int p_\theta(\mathbf{x}|\mathbf{z})\ p_\lambda(\mathbf{z})\ \mathrm{d}\mathbf{z} \\
&= \log \int \frac{q_\phi(\mathbf{z}|\mathbf{x})}{q_\phi(\mathbf{z}|\mathbf{x})}\ p_\theta(\mathbf{x}|\mathbf{z})\ p_\lambda(\mathbf{z})\ \mathrm{d}\mathbf{z} \\
&\geq \int q_\phi(\mathbf{z}|\mathbf{x})\ \log \frac{p_\theta(\mathbf{x}|\mathbf{z})\ p_\lambda(\mathbf{z})}{q_\phi(\mathbf{z}|\mathbf{x})}\ \mathrm{d}\mathbf{z} \\
&= \mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})}[\log p_\theta(\mathbf{x}|\mathbf{z})] - \mathrm{KL}[q_\phi(\mathbf{z}|\mathbf{x})||p_\lambda(\mathbf{z})]
\end{aligned}$$

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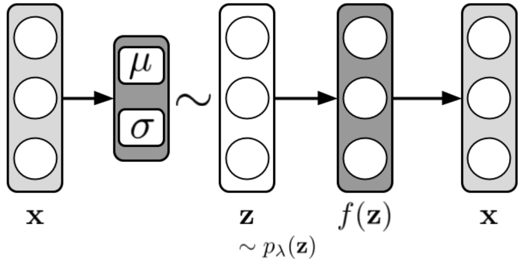

Variational posterior

$$
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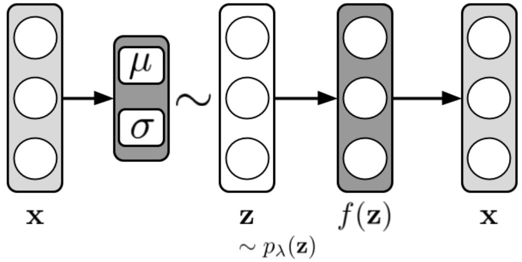

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&= \underbrace{\mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})}[\log p_\theta(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction error}} - \underbrace{\mathrm{KL}[q_\phi(\mathbf{z}|\mathbf{x})||p_\lambda(\mathbf{z})]}_{\text{Regularization}}
\end{aligned}
$$

DGM: Variational Auto-Encoder

Our objective it **the evidence lower bound.**

$$\log p(\mathbf{x}) \geq \mathbb{E}_{\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x})}[\log p_\theta(\mathbf{x}|\mathbf{z})] - \mathrm{KL}[q_\phi(\mathbf{z}|\mathbf{x})||p_\lambda(\mathbf{z})]$$

We can approximate it using **MC sample**.

$$\mathcal{L}(\mathbf{x}) \approx \frac{1}{S} \sum_{s=1}^{S} [\log p_\theta(\mathbf{x}|\mathbf{z}_s) - \log q_\phi(\mathbf{z}_s|\mathbf{x}) + \log p_\lambda(\mathbf{z}_s)]$$

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How to properly calculate gradients (*i.e.*, train the model)?

DGM: Variational Auto-Encoder

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PROBLEM: calculating gradient wrt parameters of the variational posterior (*i.e.*, sampling process).

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SOLUTION: use a non-centered parameterization (a.k.a. ***reparameterization trick***).

$$q_\phi(\mathbf{z}|\mathbf{x}) = \mathcal{N}(\mathbf{z}|\mu, \sigma^2)$$

$$\mathbf{z}_s = \mu + \sigma \odot \epsilon, \quad \epsilon \sim \mathcal{N}(0, \mathbf{I})$$

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Output of a neural network

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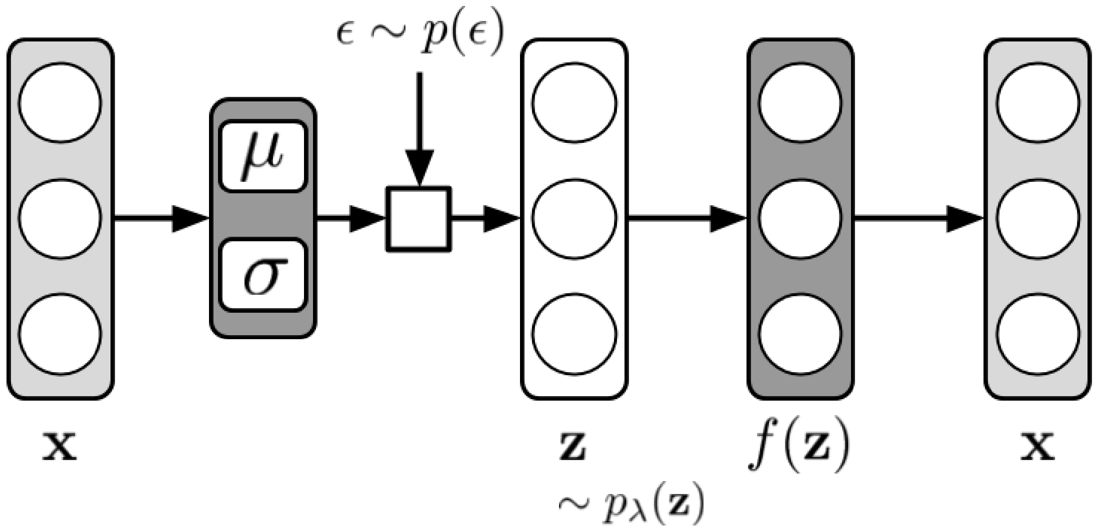
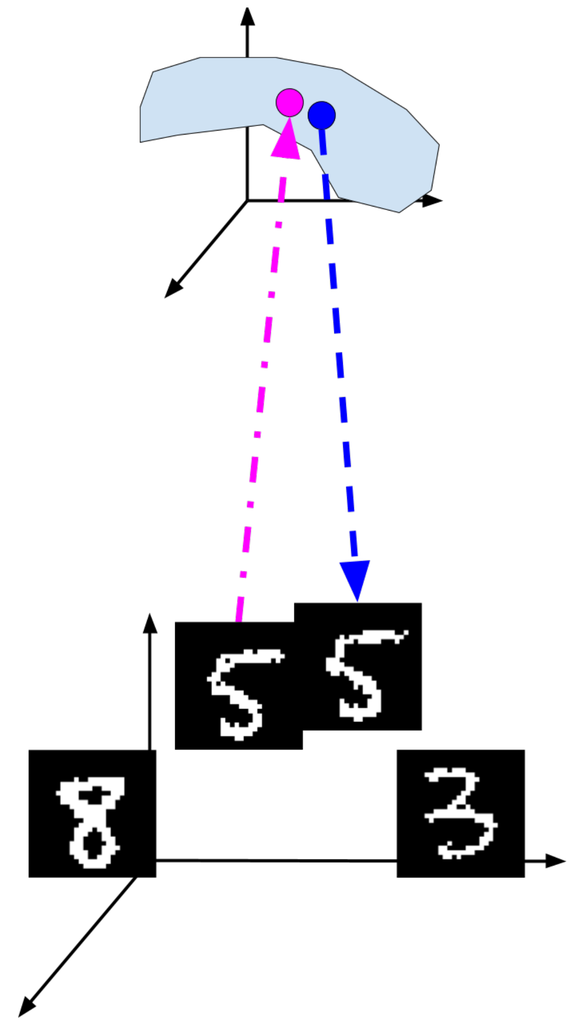
DGM: Variational Auto-Encoder





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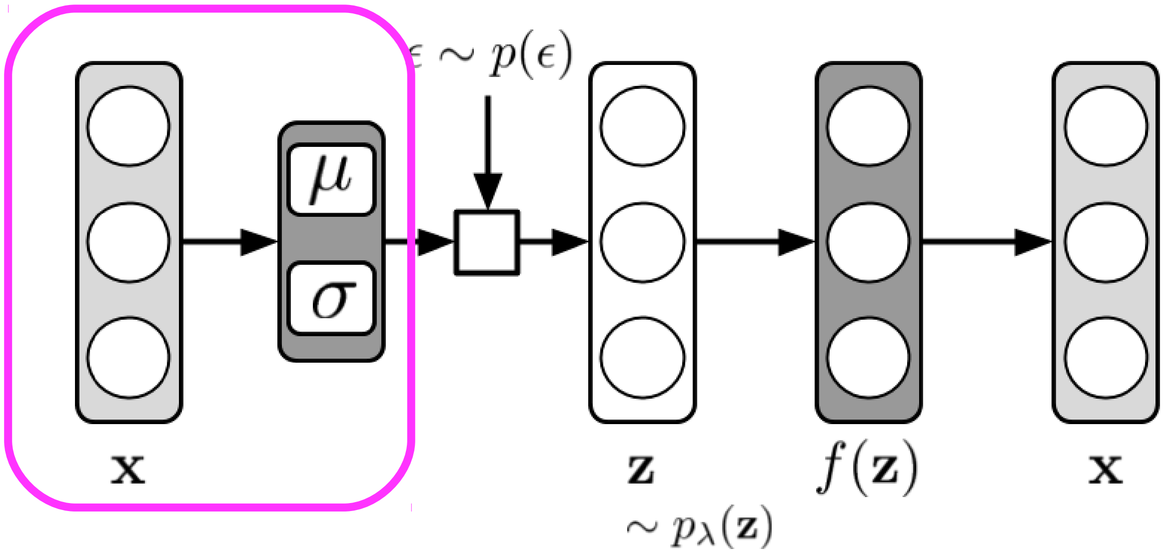
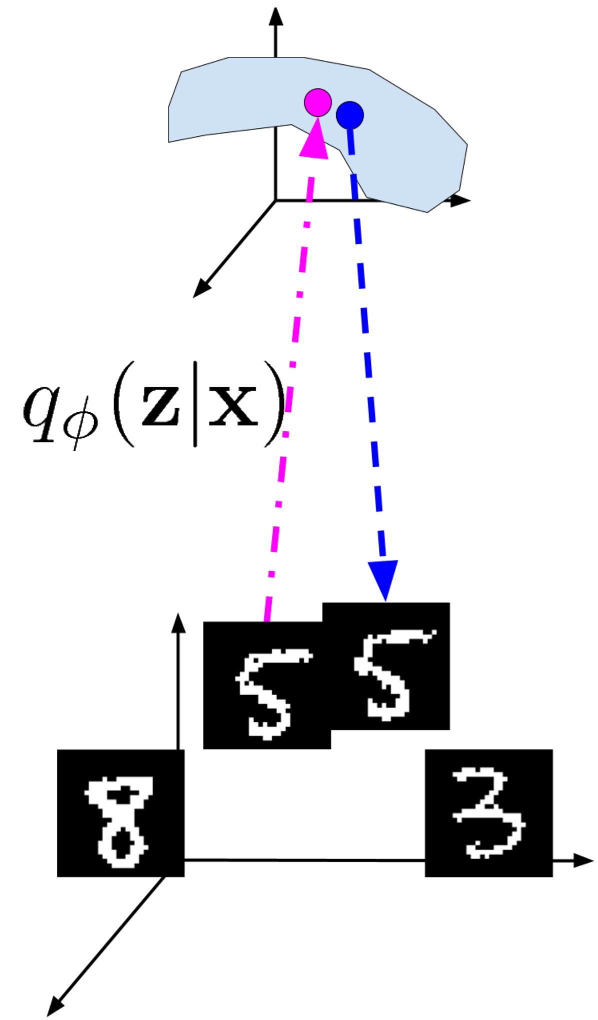
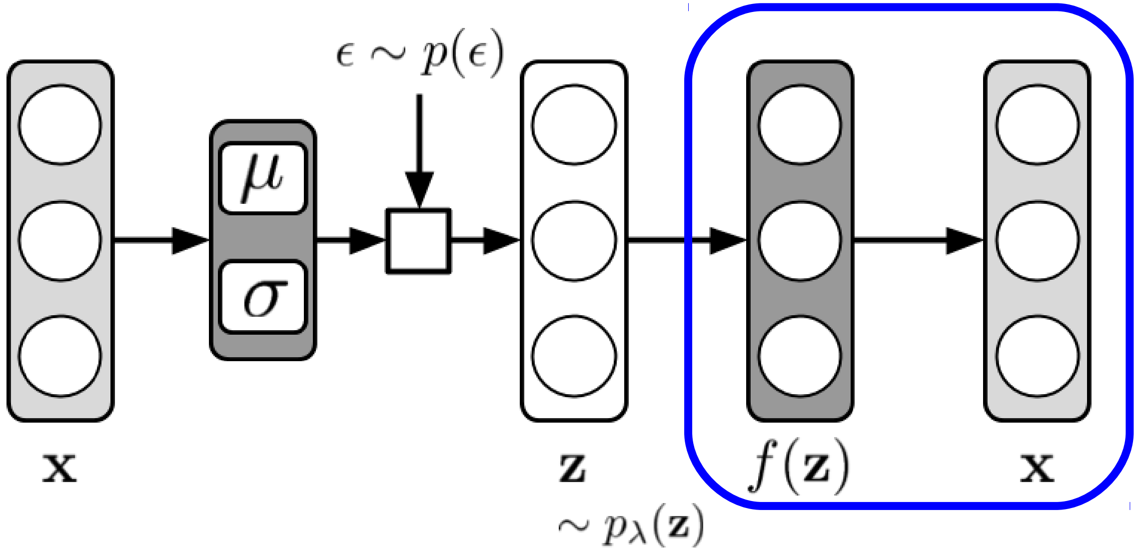


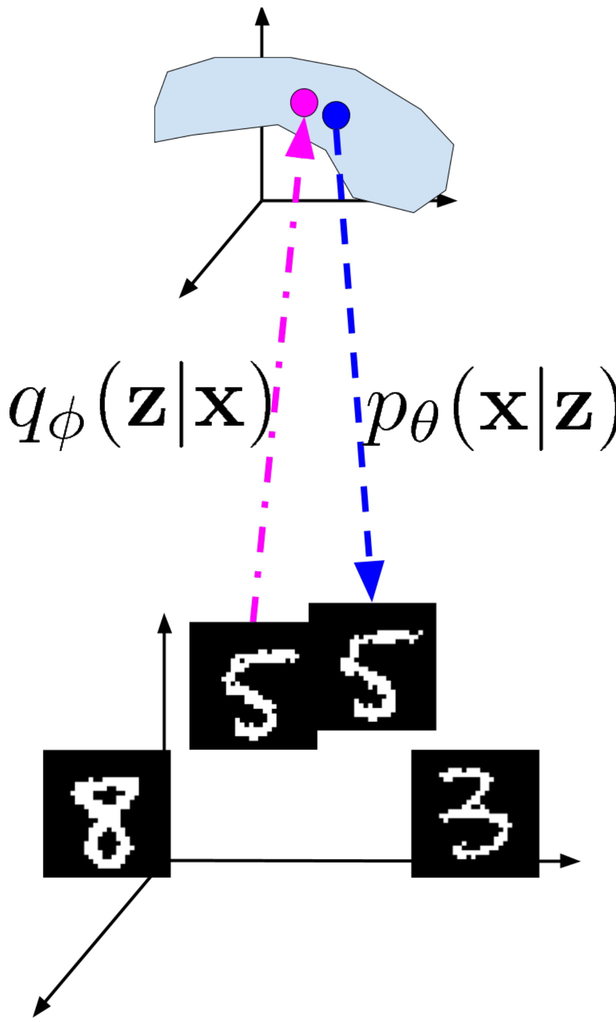


A **deep neural net** that outputs parameters of the variational posterior (**encoder**):

$$q_\phi(\mathbf{z}|\mathbf{x}) = \mathcal{N}(\mathbf{z}|\mu(\mathbf{x}), \mathrm{diag}\{\sigma^2(\mathbf{x})\})$$

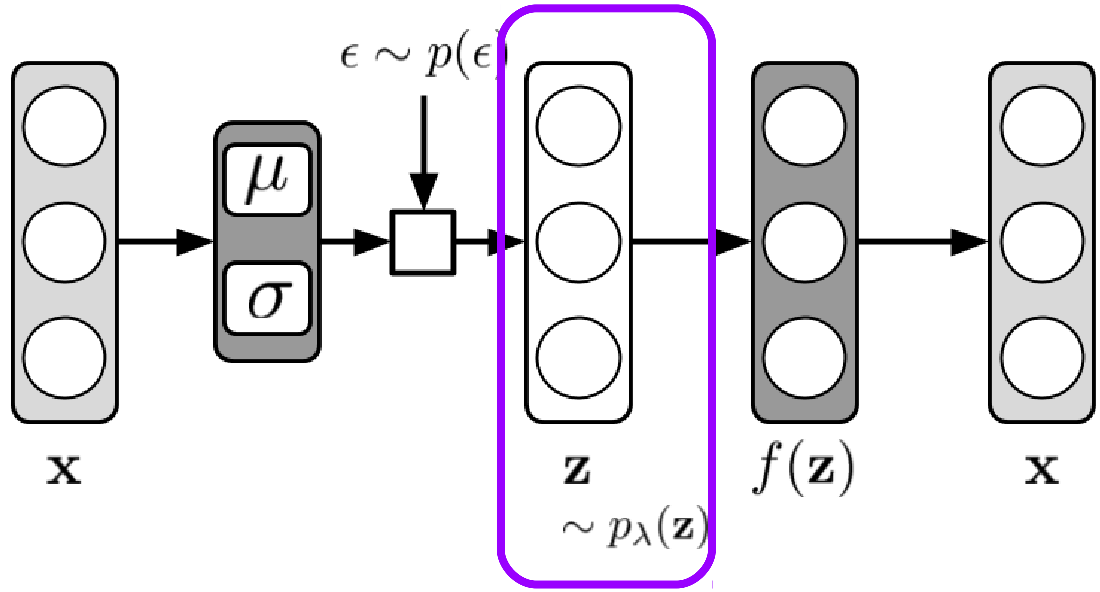
DGM: Variational Auto-Encoder

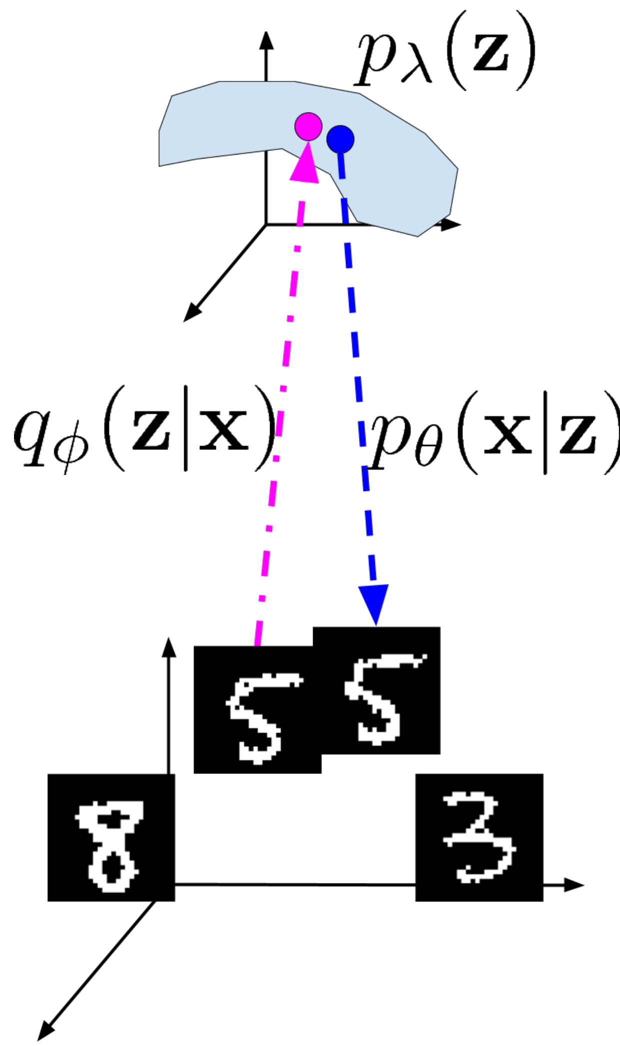




A **deep neural net** that outputs parameters of the generator (**decoder**), *e.g.*, a normal distribution or Bernoulli distribution.

DGM: Variational Auto-Encoder



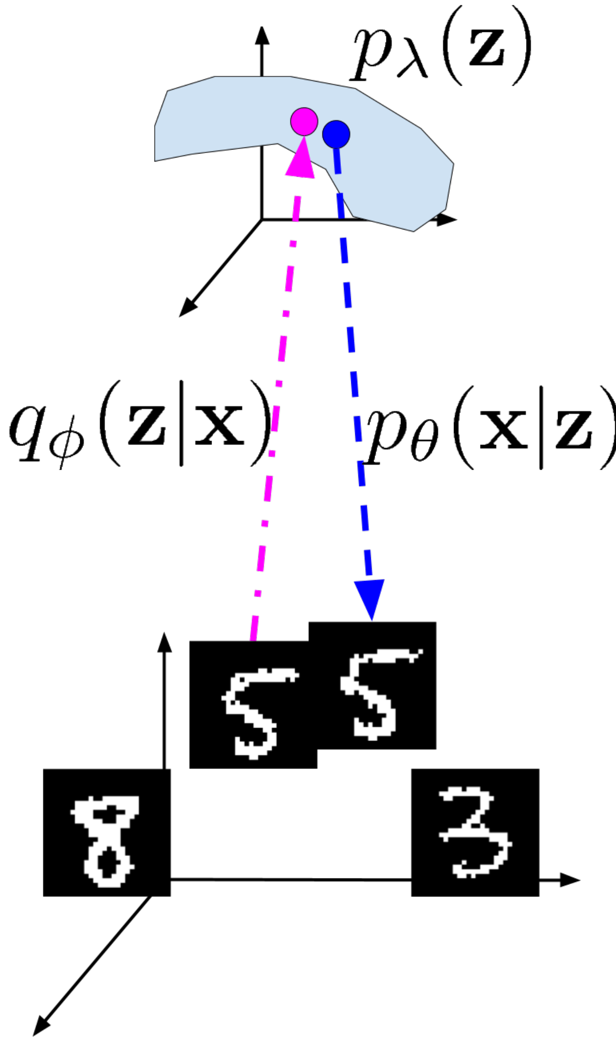


A **prior** that **regularizes** the encoder and takes part in the **generative process**.

$$p_\lambda(\mathbf{z}) = \mathcal{N}(\mathbf{z}|0, \mathbf{I})$$

DGM: Variational Auto-Encoder

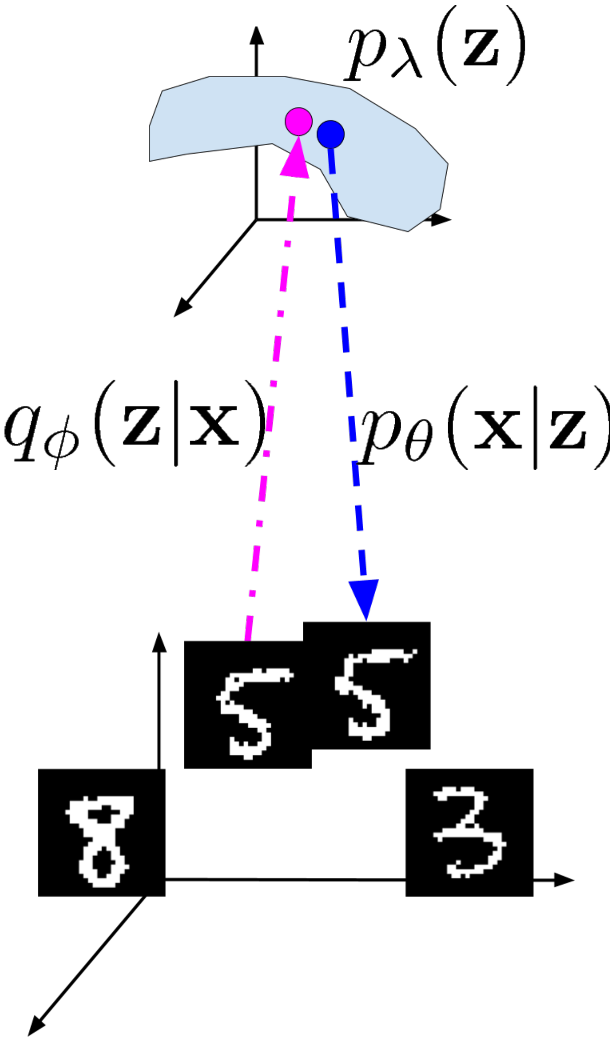
$$q_\phi(\mathbf{z}|\mathbf{x}) \propto p_\theta(\mathbf{x}|\mathbf{z})\ p_\lambda(\mathbf{z})$$



DGM: Variational Auto-Encoder

$$q_\phi(\mathbf{z}|\mathbf{x}) \propto p_\theta(\mathbf{x}|\mathbf{z})\, p_\lambda(\mathbf{z})$$

Feedforward nets
Convolutional nets
PixelCNN
Gated PixelCNN

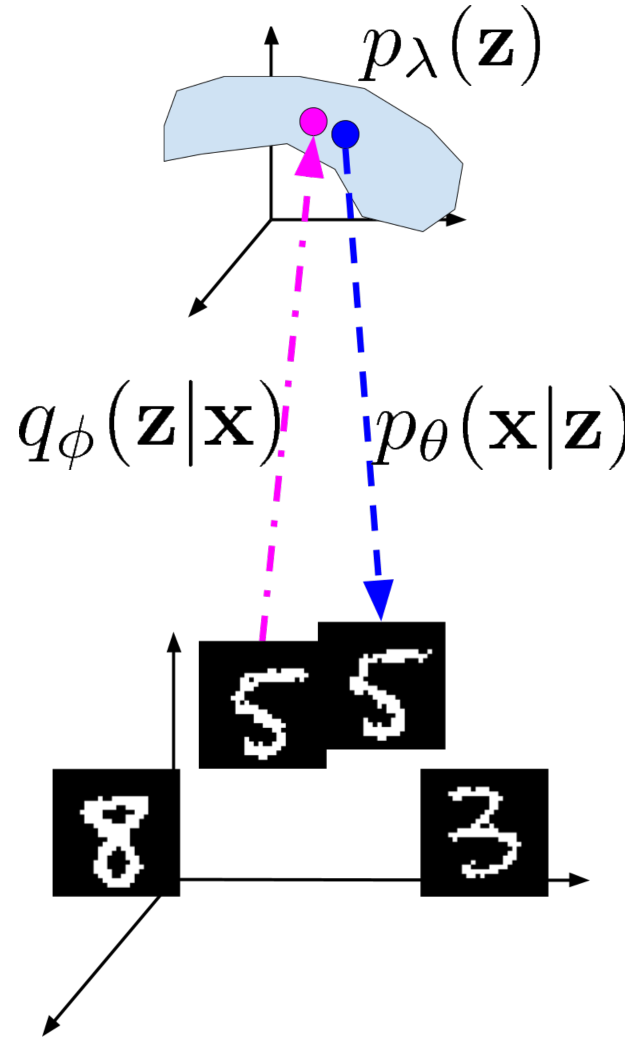


DGM: Variational Auto-Encoder

$$q_\phi(\mathbf{z}|\mathbf{x}) \propto p_\theta(\mathbf{x}|\mathbf{z})\, p_\lambda(\mathbf{z})$$

Normalizing flows
Volume-preserving flows
Gaussian processes
Stein Particle Descent
Operator VI

Feedforward nets
Convolutional nets
PixelCNN
Gated PixelCNN



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Normalizing flows
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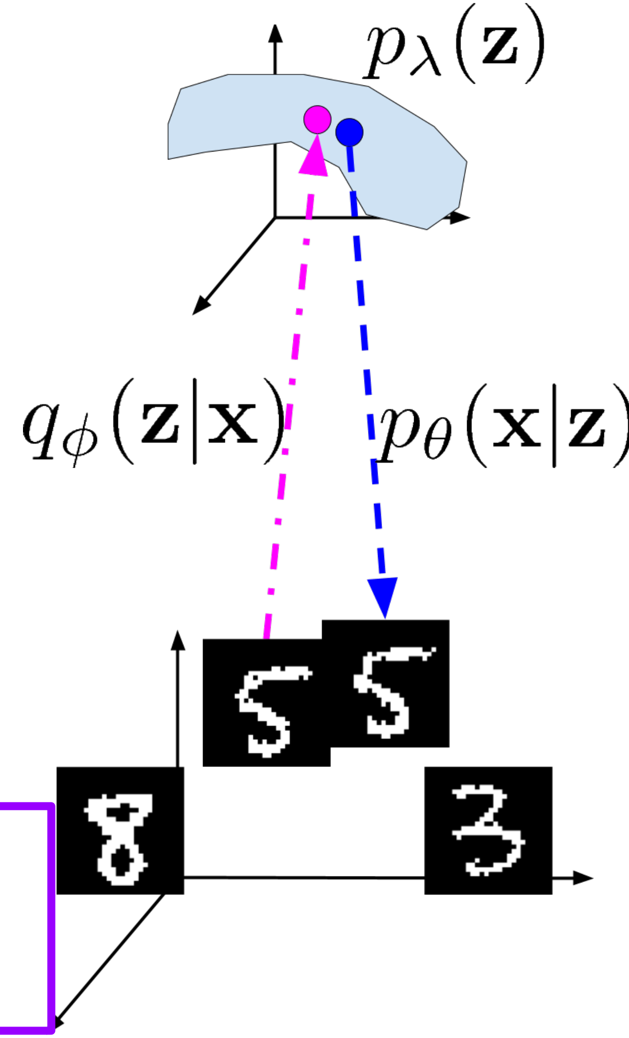
Feedforward nets
Convolutional nets
PixelCNN
Gated PixelCNN

Auto-regressive Prior
Objective Prior
Stick-Breaking Prior
VampPrior



DGM: Variational Auto-Encoder

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Normalizing flows
Volume-preserving flows
Gaussian processes
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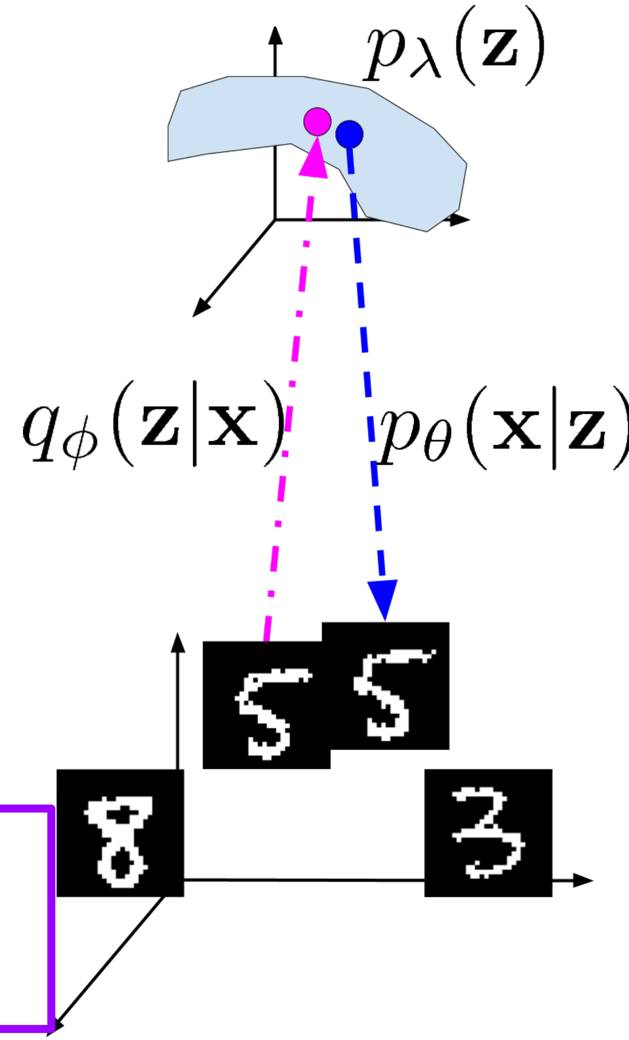
Feedforward nets
Convolutional nets
PixelCNN
Gated PixelCNN

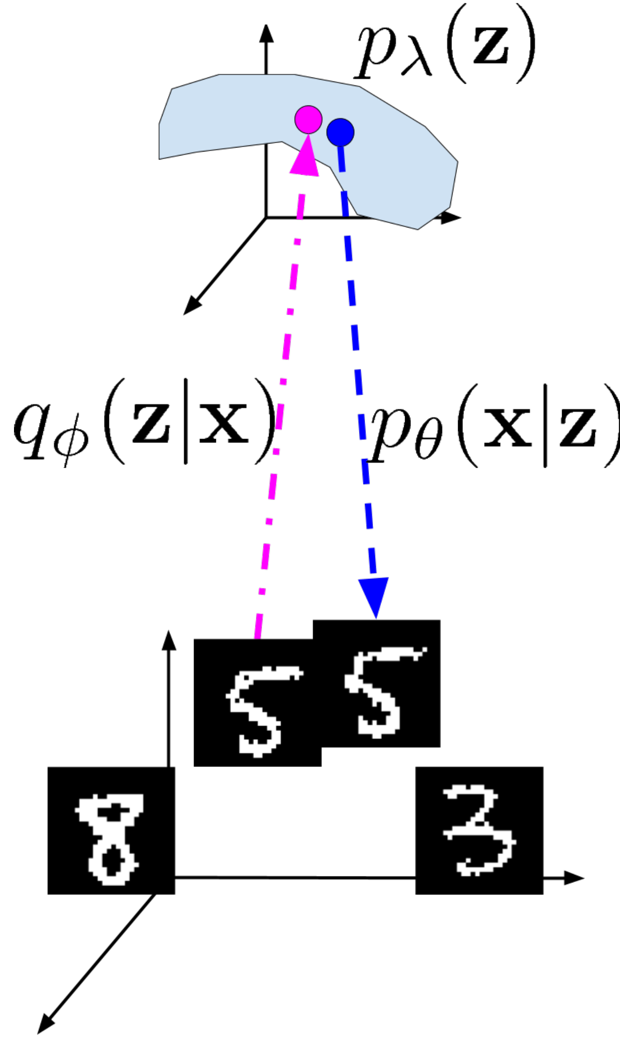
Auto-regressive Prior
Objective Prior
Stick-Breaking Prior
VampPrior

Importance Weighted AE
Renyi Divergence
Stein Divergence



Improving the posterior

$$q_\phi(\mathbf{z}|\mathbf{x}) \propto p_\theta(\mathbf{x}|\mathbf{z})\ p_\lambda(\mathbf{z})$$



Normalizing flows

- Diagonal posterior – **insufficient** and **inflexible**.

- How to get more flexible posterior?
  → apply a series of T **invertible transformations**
  $\mathbf{f}^{(t)}$ to $\mathbf{z}^{(0)} \sim q(\mathbf{z}|\mathbf{x})$.

- **New objective:**

$$\ln p(\mathbf{x}) \geq \mathbb{E}_{q(\mathbf{z}^{(0)}|\mathbf{x})}\Big[\ln p(\mathbf{x}|\mathbf{z}^{(T)}) + \sum_{t=1}^{T} \ln \Big|\det \frac{\partial \mathbf{f}^{(t)}}{\partial \mathbf{z}^{(t-1)}}\Big|\Big] - \mathrm{KL}\big(q(\mathbf{z}^{(0)}|\mathbf{x})||p(\mathbf{z}^{(T)})\big).$$

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$$\ln p(\mathbf{x}) \geq \mathbb{E}_{q(\mathbf{z}^{(0)}|\mathbf{x})}\Big[\ln p(\mathbf{x}|\mathbf{z}^{(T)}$$

$$q(\mathbf{z}^{(T)}|\mathbf{x}) = q(\mathbf{z}^{(0)}|\mathbf{x}) \prod_{t=1}^{T} \left| \det \frac{\partial f^{(t)}}{\partial \mathbf{z}^{(t-1)}} \right|^{-1} .$$

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  $\rightarrow$ apply a series of T **invertible transformations** $\mathbf{f}^{(t)}$ to $\mathbf{z}^{(0)} \sim q(\mathbf{z}|\mathbf{x})$.

- **New objective**:

$$\ln p(\mathbf{x}) \geq \mathbb{E}_{q(\mathbf{z}^{(0)}|\mathbf{x})}\left[\ln p(\mathbf{x}|\mathbf{z}^{(T)}) + \sum_{t=1}^{T} \ln \left|\det \frac{\partial \mathbf{f}^{(t)}}{\partial \mathbf{z}^{(t-1)}}\right|\right] - \mathrm{KL}\big(q(\mathbf{z}^{(0)}|\mathbf{x})||p(\mathbf{z}^{(T)})\big).$$

Rezende, D. J., & Mohamed, S. (2015). Variational inference with normalizing flows. arXiv preprint arXiv:1505.05770. ICML 2015

Normalizing flows

- Diagonal posterior – **insufficient** and **inflexible**.

- How to get more flexible posterior?

  $\rightarrow$ apply a series of T **invertible transformations**

  $\mathbf{f}^{(t)}$ to $\mathbf{z}^{(0)} \sim q(\mathbf{z}|\mathbf{x})$.

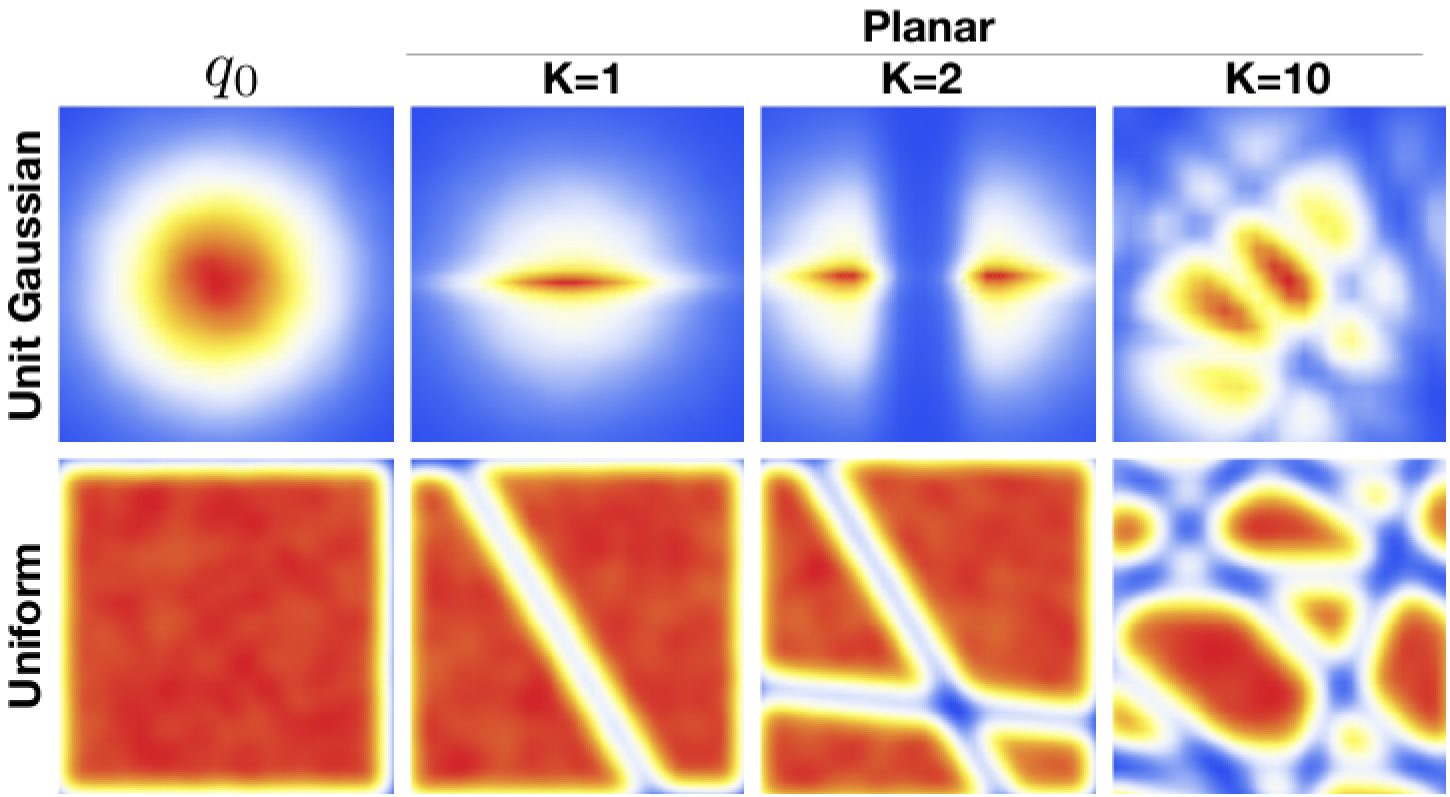
- **New objective**:

$$\ln p(\mathbf{x}) \geq \mathbb{E}_{q(\mathbf{z}^{(0)}|\mathbf{x})}\left[\ln p(\mathbf{x}|\mathbf{z}^{(T)}) + \sum_{t=1}^{T} \ln \left|\det \frac{\partial \mathbf{f}^{(t)}}{\partial \mathbf{z}^{(t-1)}}\right|\right] - \mathrm{KL}\big(q(\mathbf{z}^{(0)}|\mathbf{x})||p(\mathbf{z}^{(T)})\big).$$

Jacobian-determinant: (i) **general normalizing flow** (|det J| is ***easy*** to compute)

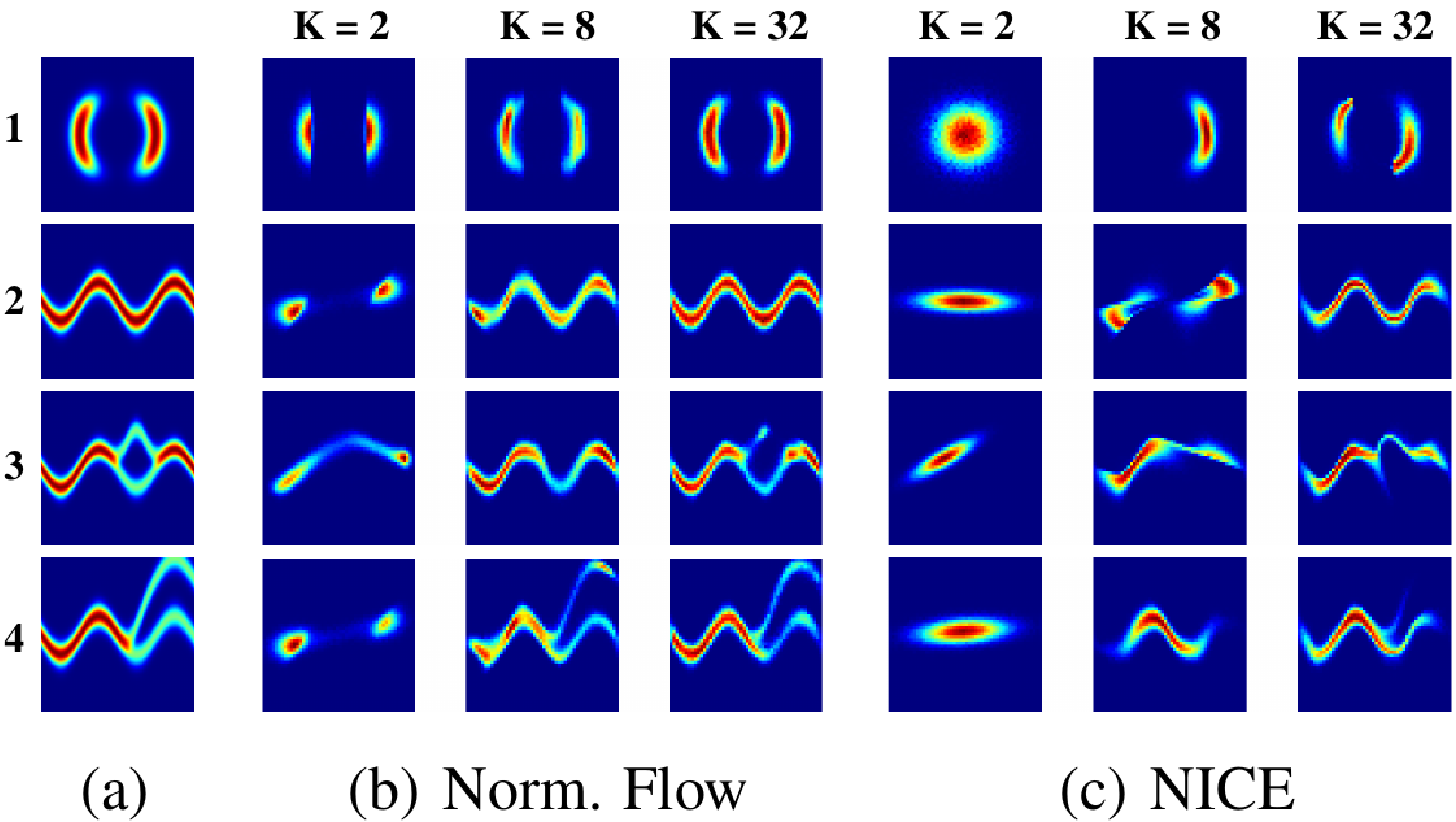
(ii) **volume-preserving flow**, *i.e.*, |det J|=1

Normalizing Flow



Rezende, D. J., & Mohamed, S. (2015). Variational inference with normalizing flows. arXiv preprint arXiv:1505.05770. ICML 2015

Normalizing Flow



Rezende, D. J., & Mohamed, S. (2015). Variational inference with normalizing flows. arXiv preprint arXiv:1505.05770. ICML 2015

Extensions of normalizing flows

- How to obtain more **flexible** posterior and preserve **$|\det \mathrm{J}|=1$**?
  - using ***orthogonal matrices*** → **Householder flow**

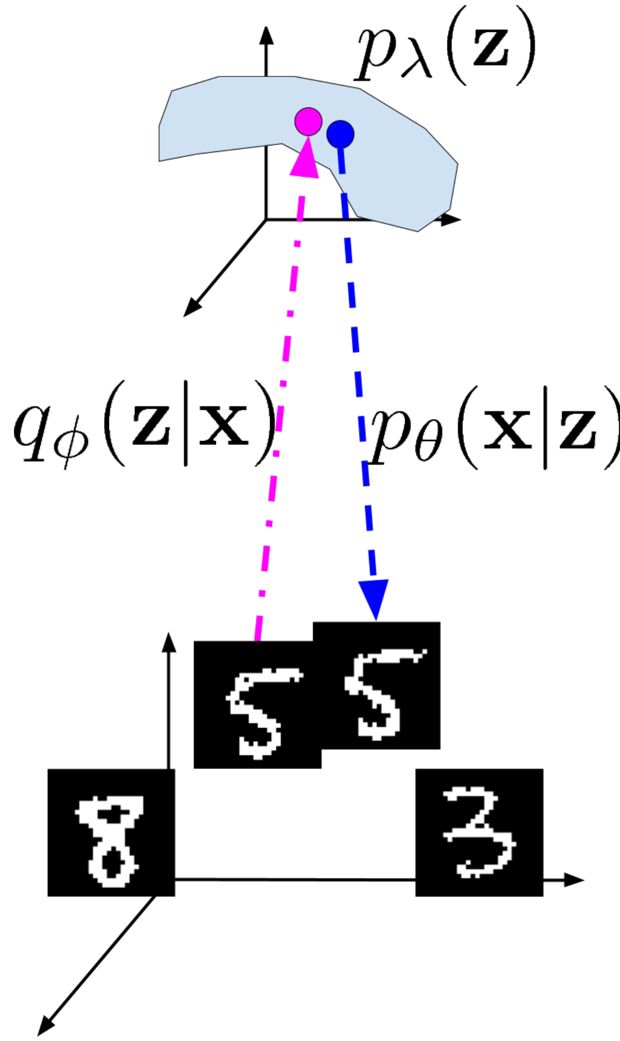
    Tomczak, J. M., & Welling, M. (2016). Improving Variational Inference with Householder Flow. arXiv preprint arXiv:1611.09630. NIPS Workshop on Bayesian Deep Learning 2016

- **General** normalizing flow:
  - using ***autoregressive model*** → **Inverse Autoregressive Flow**

    Kingma, D. P., Salimans, T., Jozefowicz, R., Chen, X., Sutskever, I., & Welling, M. (2016). Improving Variational Inference with Inverse Autoregressive Flow. NIPS 2016

Improving the decoder

$$q_\phi(\mathbf{z}|\mathbf{x}) \propto p_\theta(\mathbf{x}|\mathbf{z}) p_\lambda(\mathbf{z})$$

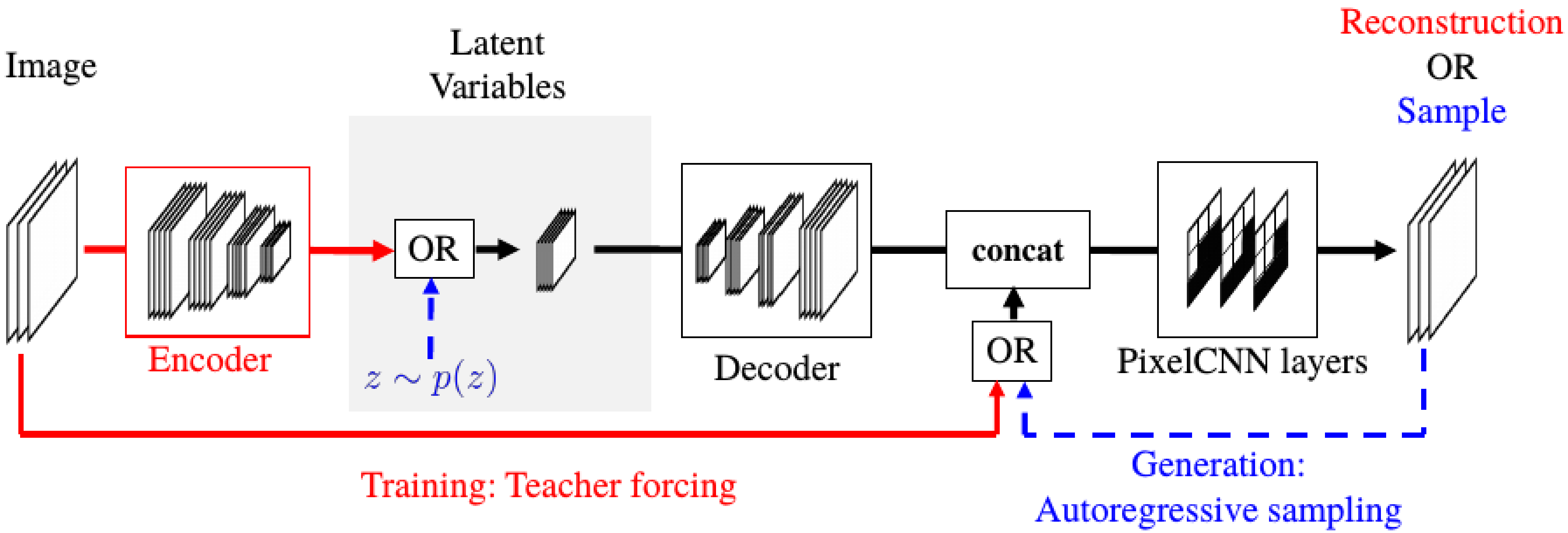


Improving the decoder

- Dependency only on $\mathbf{z}$ – **missing correlations**.
- How to get more flexible decoderposterior?
  $\rightarrow$ apply **autoregressive model**

$$p(\mathbf{x}|\mathbf{z}) = p(x_1|\mathbf{z}) \prod_{d=2}^{D} p(x_d|x_1, \ldots, x_{d-1}, \mathbf{z})$$

PixelVAE (PixelCNN + VAE)



Gulrajani, I., Kumar, K., Ahmed, F., Taiga, A. A., Visin, F., Vazquez, D., & Courville, A. (2016). PixelVAE: A latent variable model for natural images. arXiv preprint arXiv:1611.05013.

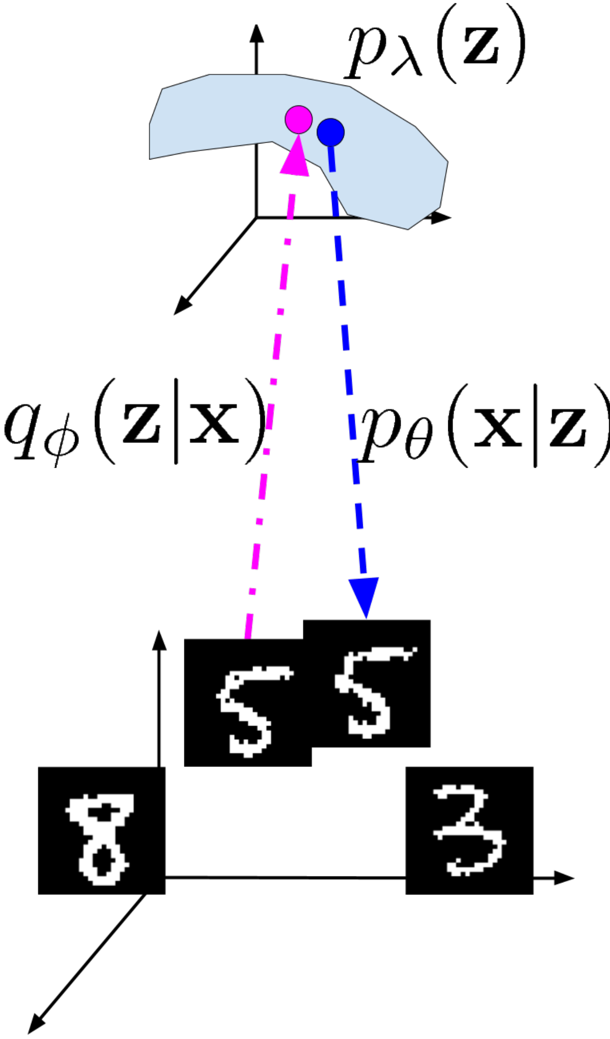
PixelVAE (PixelCNN + VAE)

Figure 6: Samples from hierarchical PixelVAE on the 64x64 ImageNet dataset.

Gulrajani, I., Kumar, K., Ahmed, F., Taiga, A. A., Visin, F., Vazquez, D., & Courville, A. (2016). PixelVAE: A latent variable model for natural images. arXiv preprint arXiv:1611.05013.

Improving the prior

$$q_\phi(\mathbf{z}|\mathbf{x}) \propto p_\theta(\mathbf{x}|\mathbf{z})\ p_\lambda(\mathbf{z})$$



Improving the prior

- Standard normal prior – **unimodal, too restrictive**.

- How to get more flexible prior?

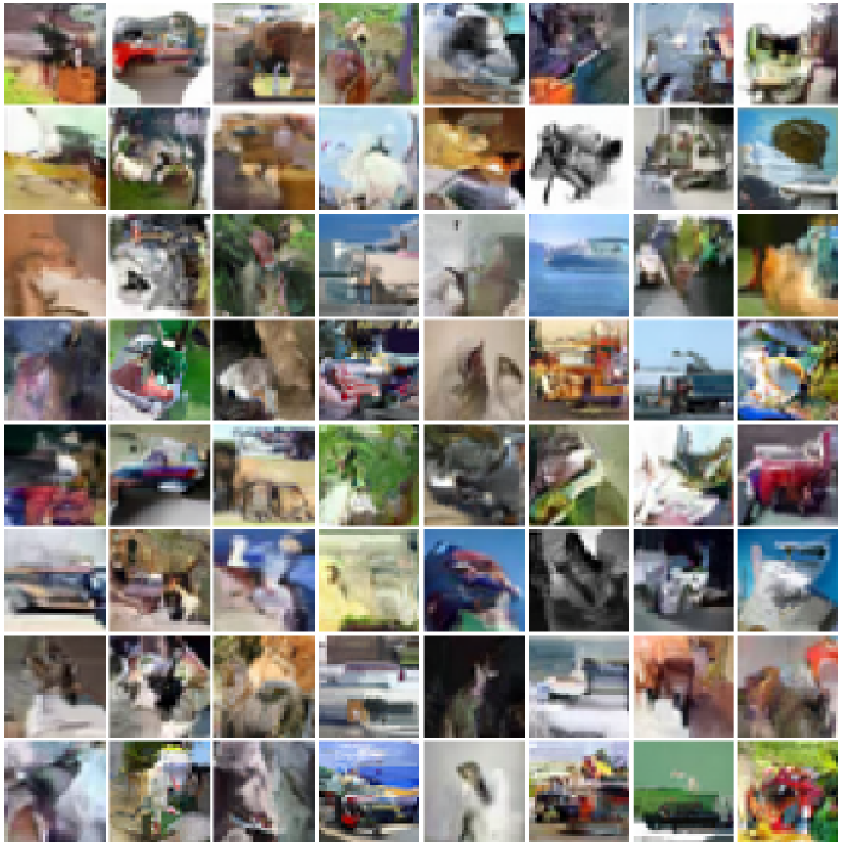
  $\rightarrow$ apply **autoregressive prior**

Chen, X., Kingma, D. P., Salimans, T., Duan, Y., Dhariwal, P., Schulman, J., ... & Abbeel, P. (2016). Variational lossy autoencoder. arXiv preprint arXiv:1611.02731.

  $\rightarrow$ apply **variational mixture of posteriors (VampPrior)**

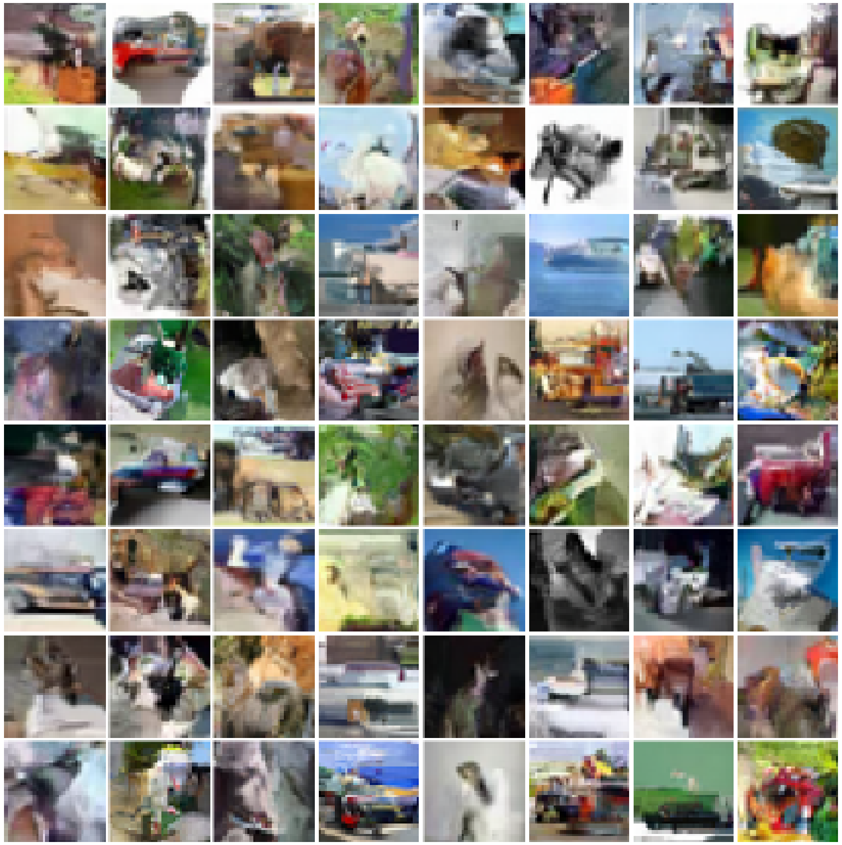
Tomczak, J. M., & Welling, M. (2017). VAE with a VampPrior. arXiv preprint arXiv:1705.07120.

Autoregressive prior



Chen, X., Kingma, D. P., Salimans, T., Duan, Y., Dhariwal, P., Schulman, J., ... & Abbeel, P. (2016). Variational lossy autoencoder. arXiv preprint arXiv:1611.02731.

Autoregressive prior



Chen, X., Kingma, D. P., Salimans, T., Duan, Y., Dhariwal, P., Schulman, J., ... & Abbeel, P. (2016). Variational lossy autoencoder. arXiv preprint arXiv:1611.02731.

VampPrior

(a) real data (b) VAE (c) HVAE + VampPrior (d) convHVAE + VampPrior

Tomczak, J. M., & Welling, M. (2017). VAE with a VampPrior. arXiv preprint arXiv:1705.07120.

Some extensions and applications of VAE

- Semi-supervised learning with VAE.

  Kingma, D. P., Mohamed, S., Rezende, D. J., & Welling, M. (2014). *Semi-supervised learning with deep generative models*. NIPS

- VAE for sequences.

  Bowman, S. R., Vilnis, L., Vinyals, O., Dai, A. M., Jozefowicz, R., & Bengio, S. (2015). *Generating sentences from a continuous space*. arXiv preprint arXiv:1511.06349.

  Chung, J., Kastner, K., Dinh, L., Goel, K., Courville, A. C., & Bengio, Y. (2015). *A recurrent latent variable model for sequential data*. NIPS

- More powerful decoders (using PixelCNN).

  Gulrajani, I., Kumar, K., Ahmed, F., Taiga, A. A., Visin, F., Vazquez, D., & Courville, A. (2016). *PixelVAE: A latent variable model for natural images*. arXiv preprint arXiv:1611.05013.

  Chen, X., Kingma, D. P., Salimans, T., Duan, Y., Dhariwal, P., Schulman, J., ... & Abbeel, P. (2016). *Variational lossy autoencoder*. arXiv preprint arXiv:1611.02731.

Some extensions and applications of VAE

- Applications: graph data

Kipf, T. N., & Welling, M. (2016). *Variational Graph Auto-Encoders*. arXiv preprint arXiv:1611.07308. NIPS Workshop

Berg, R. V. D., Kipf, T. N., & Welling, M. (2017). *Graph Convolutional Matrix Completion*. arXiv preprint arXiv:1706.02263.

- Applications: drug response prediction

Rampasek, L., & Goldenberg, A. (2017). *Dr.VAE: Drug Response Variational Autoencoder*. arXiv preprint arXiv:1706.08203.

- Applications: text generation

Yang, Z., Hu, Z., Salakhutdinov, R., & Berg-Kirkpatrick, T. (2017). *Improved Variational Autoencoders for Text Modeling using Dilated Convolutions*. arXiv preprint arXiv:1702.08139.

DGM: VAE

PROS

Log-likelihood framework

Easy **sampling**

Training using **gradient-based methods**

Stable training

Discovers **latent representation**

Could be easily combined with other probabilistic frameworks

CONS

Only **explicit** models

Produces **blurry images(?)**

$$1283 + 1146$$

Number of citations of seminal papers on GANs and VAE.*

**According to GoogleScholar, 26.09.2017*

In order to *make better* ***decisions***, we need a *better understanding* of ***reality***.

=

generative modeling

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https://jmtomczak.github.io

Code on github:
https://github.com/jmtomczak

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RESEARCH & INNOVATION
Marie Skłodowska-Curie actions